

Equations and Inequalities Problem Set

1. [AJC Prelims 17]

Mr Tan invested a total of \$25,000 in a structured deposit account, bonds and an estate fund. He invested \$7,000 more in bonds than in estate fund. The projected annual interest rates for structured deposit account, bonds and estate fund are 2%, 3% and 4.5% respectively. Money that is not drawn out at the end of the year will be re-invested for the following year.

Mr Tan plans to draw out his money from all investments at the end of the second year and estimates that he will receive a total of \$26,300. Find the amount of money Mr Tan invested in each investment, giving your answer to the nearest dollar. [5]

2. [CJC Prelims 17]

The curve with equation $y = f(x)$, where $f(x)$ is a cubic polynomial, has a maximum point with coordinates $(-2, \frac{34}{3})$ and a minimum point with coordinates $(3, -\frac{19}{2})$. Find the equation of the curve. [4]

3. [DHS Prelims 17]

Using an algebraic method, find the set of values of x that satisfies the inequality

$$2 - x \leq \frac{x}{2 - x}. \quad [3]$$

Hence solve

$$2 - x^2 \leq \frac{x^2}{2 - x^2}. \quad [2]$$

4. [DHS Prelims 17]

For this question, leave your answers to the nearest dollar.

Mr Foo invested \$25,000 in three different stocks A , B and C . After a year, the value of the stocks A and B grew by 2% and 6% respectively, while the value of stock C fell by 2%. Mr Foo did not gain or lose any money. Let a , b and c denote the amount of money he invested in stocks A , B and C respectively.

(a) Find expressions for a and b , in terms of c . [2]

(b) Find the values between which c must lie. [2]

5. [HCI Prelims 17]

- (a) By first expressing $3x - x^2 - 4$ in completed square form, show that $3x - x^2 - 4$ is always negative for all real values of x . [2]
- (b) Hence or otherwise, without the use of a calculator, solve this inequality

$$\frac{(3x - x^2 - 4)(x - 1)^2}{x^2 - 2x - 5} \leq 0,$$

leaving your answer in exact form. [4]

6. [IJC Prelims 17]

Without using a graphic calculator, solve the inequality

$$\frac{4x^2 + 7x + 1}{3x + 1} \leq x + 2.$$

[3]

Hence solve the inequality

$$\frac{4x + 7\sqrt{x} + 1}{3\sqrt{x} + 1} \leq \sqrt{x} + 2.$$

[2]

7. [TPJC Prelims 17]

Without using a calculator, solve the inequality

$$\frac{3x^2 + 7x + 1}{x + 3} < 2x - 1.$$

[4]

8. [MI Prelims 17]

The sum of the first n terms of a sequence is denoted by S_n . The first term of the sequence is 3 and it is known that $S_3 = 21$ and $S_{10} = 210$. Given that S_n is a quadratic polynomial in n , find S_n in terms of n . [3]

Answers

1. $x = 13938, y = 9031, z = 2031$.
2. $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$.
3. $\{x : 1 \leq 2 < 2 \text{ or } x \geq 4\}$.
 $x \leq -2 \text{ or } -\sqrt{2} < x \leq -1 \text{ or } 1 \leq x < \sqrt{2} \text{ or } x \geq 2$.
4. (a) $a = 37500 - 2c, b = c - 12500$.
(b) Between 12500 and 18750.
5. $x < 1 - \sqrt{6} \text{ or } x > 1 + \sqrt{6} \text{ or } x = 1$.
6. $x \leq -1 \text{ or } -\frac{1}{3} < x \leq 1$.
 $0 \leq x \leq 1$.
7. $x < -3$.
8. $S_n = 2n^2 + n$.