

Integration Techniques Problem Set

1. [ACJC Prelims 17]

Show that for any real constant k ,

$$\int t^2 e^{-kt} dt = -e^{-kt} \left(\frac{a}{k} t^2 + \frac{b}{k^2} t + \frac{c}{k^3} \right) + D,$$

where D is an arbitrary constant, and a, b and c are constants to be determined. [3]

2. [ACJC Prelims]

(a) Find $\int \frac{x}{(1+x^2)^2} dx.$ [2]

(b) By using the substitution $x = \tan \theta$, show that

$$\int \frac{1}{(1+x^2)^2} dx = k \left(\frac{x}{1+x^2} + \tan^{-1} x \right) + c,$$

where c is an arbitrary constant and k is a constant to be determined. [5]

(c) Hence find $\frac{x^2}{(1+x^2)^2} dx.$ [3]

(d) Using all of the above, find $\int \frac{x^2 + 2x + 5}{(1+x^2)^2} dx$, simplifying your answer. [2]

3. [AJC Prelims 17]

Show by integration that

$$\int e^{-2x} \sin x dx = -\frac{2}{5} e^{-2x} \sin x - \frac{1}{5} e^{-2x} \cos x + A$$

where A is an arbitrary constant. [3]

4. [DHS Prelims 17]

(a) Find $\frac{d}{dx}(\tan^2)x.$ Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \tan x e^{\tan^2 x} dx$, leaving your answer in exact form. [3]

(b) By expressing $1 + 72x - 32x^3$ as $1 + mx(9 - 4x^2)$ where m is a constant, find $\int \frac{1 + 72x - 32x^3}{\sqrt{9 - 4x^2}} dx.$ [2]

5. [HCl Prelims 17]

By writing $\sec^3 x = \sec x \sec^2 x$, find $\int \sec^3 x \, dx$.

Hence find the exact value of $\int_0^{\tan^{-1} 2} \sec^3 x \, dx$. [6]

6. [IJC Prelims 17]

(a) Find $\int n \cos^{-1}(nx) \, dx$, where n is a positive constant. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{2n}} n \cos^{-1}(nx) \, dx$. [2]

7. Find

(a) $\int \frac{1}{16x^2 + 9} \, dx$. [2]

(b) $\int \frac{x}{16x^2 + 9} \, dx$. [2]

Hence find $\int \frac{16x^3 - 5}{16x^2 + 9} \, dx$. [3]

8. Find

(a) $\int \frac{1}{x^2 + 2x - 2} \, dx$. [2]

(b) $\int \cos 5x \cos 2x \, dx$. [3]

(c) $\int \sin^2 2x \, dx$. [2]

Answers

1. $-\mathrm{e}^{-kt}\left(\frac{1}{k}t^2 + \frac{2}{k^2}t + \frac{2}{k^3}\right) + D.$
2. (a) $\frac{-1}{2(1+x^2)} + c.$
(b) $\frac{1}{2}\left(\frac{x}{1+x^2} + \tan^{-1} x\right) + c.$
(c) $3\tan^{-1} x + \frac{2x-1}{1+x^2} + c.$
3. (a) $\frac{\mathrm{e}-1}{2}.$
(b) $\frac{1}{2}\sin^{-1} \frac{2x}{3} - \frac{2}{3}(9 - 4x^2)^{\frac{3}{2}} + C.$
4. $\sqrt{5} + \frac{1}{2}\ln(\sqrt{5} + 2).$
5. (a) $(nx)\cos^{-1}(nx) - \sqrt{1 - n^2x^2} + C.$
(b) $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1.$
6. $\frac{1}{2}(\mathrm{e}^x \sin x - \mathrm{e}^x \cos x) + D.$
7. (a) $\frac{1}{12}\tan^{-1} \frac{4x}{3} + C.$
(b) $\frac{1}{32}\ln(16x^2 + 9) + D.$
(c) $\frac{x^2}{2} - \frac{9}{32}\ln(16x^2 + 9) - \frac{5}{12}\tan^{-1} \frac{4x}{3} + E.$
8. (a) $\frac{1}{2\sqrt{3}}\ln \left| \frac{x-1+\sqrt{3}}{x-1-\sqrt{3}} \right| + C.$
(b) $\frac{1}{2}\left(\frac{\sin 7x}{7} + \frac{\sin 3x}{3}\right) + D.$
(c) $\frac{x}{2} - \frac{\sin 4x}{8} + E.$