Sampling Theory and Hypothesis Testing

1. **[CJC 14 Prelims]**
   The mass of a bar of chocolate is normally distributed with mean μ grams and standard deviation σ grams. Given that 3% of the chocolates have a mass less than 42 grams and 3% have a mass more than 48 grams,

   (a) state the value of μ and [1]

   (b) show that $\sigma = 1.5951$, correct to 5 significant figures. [1]

   (c) A random sample of 70 bars of chocolate is taken. What is the probability that the sample mean lies between 43 grams and 45 grams? [4]

2. **[JJC 14 Prelims]**
   The staff of a hospital emergency unit observes that, on average, the proportion of the admitted patients who require surgery is 40%.

   State the mean and variance of the number of patients who require surgery in a random sample of 15 patients.

   In a study conducted over a period of time, 50 such samples of 15 patients are randomly selected. Find the probability that the mean number of admitted patients that require surgery is more than 5.5. [4]

3. **[NJC 14 Prelims]**
   The weight in kg of a whole chicken sold in "Shop and Pay" is a random variable with a distribution $N(2.2, 0.5^2)$.

   The mean weight of n randomly chosen chickens is denoted by $\overline{C}$ kg. Given that $P(\overline{C} > 2.35) = 0.0502$, find the value of n. [3]

4. **[SRJC 14 Prelims]**
   In a particular high school, 94% of its students own a Friend Book account. A random sample of 80 students is taken from the high school. The random variable X denotes the number of students in the sample who own a Friend Book account.

   (a) State, in the context of this question, an assumption needed to model this situation by a binomial distribution. [1]

   (b) If the sample has at least 70 students who own a Friend Book account, find the probability that there are at most 74 students who own a Friend Book account in the sample. [2]

   (c) Estimate the probability that there are exactly 75 students who own a Friend Book account in the sample. [3]

(d) Sixty samples each of size 80 are taken from the high school. Find the probability that the average number of students who own a Friend Book account in each sample exceeds 76. [2]

5. **[CJC 14 Prelims]**
A company manufactures bricks. The random variable of the mass of a brick is denoted by X grams. The masses of a random sample of 8 bricks are summarized by

$$\sum x = 8\ 293, \qquad \sum x^2 = 8\ 637\ 150.$$

(a) Find unbiased estimates of the population mean and variance. [2]

The mean mass of a brick is claimed to be 1000 grams.

(b) Stating a necessary assumption, carry out a test at the 5% significance level on whether the mean mass of the bricks has increased. [5]

(c) Suppose now that the mass of bricks follows a normal distribution with a known population variance. What change would there be in carrying out the test using the same sample? [1]

Another company producing bricks claims that the mass of a brick has a mean μ_0 grams. The mass of bricks is known to have a normal distribution with standard deviation 75 grams. A random sample of 80 bricks is selected and the mean is 1026 grams.

Find, at the 5% level of significance, the least value of μ_0, correct to the nearest gram, for which there is sufficient evidence for the company to have overstated the mean mass. [4]

6. **[JJC 14 Prelims]**
The number of packets of cereal sold in a supermarket is a normally distributed random variable. Over a long period it is known that the mean number of packets of cereal sold per day is 124.5. Following a promotional campaign on television advertisement, the daily sales, x packets, is measured for a random sample of 12 days. The results are summarized as follows:

$$\sum x = 1620, \qquad \sum x^2 = 221\ 175.$$

(a) Find unbiased estimates of the population mean and variance. [2]

(b) Test, at the 1% level of significance, whether the mean number of packets of cereal sold per day has changed. [4]

After some modifications to the promotional campaign, a new sample of 12 days is taken and the population standard deviation is known to be 13. The owner of the supermarket claims that the mean daily sales has increased from 124.5.

Find the range of values of the mean daily sales of the new sample if the owner's claim is to be accepted at the 1% level of significance. [4]

Explain what you understand by the expression 'at the 1% level of significance' in the context of this question. [1]

7. **[MI 14 Prelims]**
   A machine is used to pack rice into packets of 10 kg. After a few years, the owner started receiving complaints that the mean weight of rice in a packet is not 10 kg. The owner starts to suspect that the machine is faulty due to wear and tear.

   He then proceeds to weigh the amount of rice in 13 packets. The results are summarized below:

   $$\sum(x - 10) = 2.6, \qquad \sum(x - 10)^2 = 0.9$$

   where x denotes the weight of rice in one packet in kg.

   (a) Calculate the unbiased estimates for the mean and variance of the weight of rice in one packet. [2]

   (b) Test at the 5% significance level if the owner's suspicion is justifiable, stating a necessary assumption to conduct the test. [5]

   (c) The same test in part (b) was conducted but at a different significance level $\alpha\%$. State the least value of α so that the null hypothesis is rejected. [1]

8. **[SRJC 14 Prelims]**

   (a) A famous zoologist Elsa claims that the mean tail length of Proboscis Monkeys is at most 65 cm on a particular remote island. The tails of a random sample of 20 Proboscis Monkeys are measured and found to have mean 65.5 cm and standard deviation 0.9 cm.

   Test at the 1% significance level whether Elsa's claim is valid. [5]

(b) Another famous zoologist Anna claims that the mean tail length of Proboscis Monkeys on another island is 63 cm. The tails of a new random sample of 25 Proboscis Monkeys on the island have been measured and the mean length of these monkeys is found to be t cm.

 i. Assuming that the tail lengths of Proboscis Monkeys on the island are normally distributed with standard deviation 5.8 cm, find the set of values of t for Anna's claim to be rejected at the 5% level of significance. [4]

 ii. Explain the term "5% level of significance" in the context of the question. [1]

Answers

Answers may have mistakes.

1. (a) $\mu = 45$.
   (c) 0.500.

2. 0.969.

3. $n = 30$.

4. (a) The event that students owning a Friend Book account are independent of one another.
   (b) 0.342.
   (c) 0.175.
   (d) 0.00177.

5. (a) Unbiased estimate of population mean, $\mu = \overline{x} = 1040$.
   Unbiased estimate of population variance, $\sigma^2 = s^2 = 5770$.
   (b) p-value $= 0.108 > 0.1$. We do not reject H_0. There is insufficient evidence at 5% level of significance to indicate the company's claim is not valid.
   (c) A Z-test will be used instead of a t-test.

   Least $\mu_0 = 1040$.

6. (a) Unbiased estimate of population mean, $\mu = \overline{x} = 135$.
   Unbiased estimate of population variance, $\sigma^2 = s^2 = 225$.
   (b) p-value $= 0.0337 > 0.01$. We do not reject H_0. There is insufficient evidence at 1% level of significance to indicate that the mean number of packets of cereal sold per day has changed.

   $\overline{x} \geq 133$.
   It means that there is a probability of 0.01 of our hypothesis test concluding that the mean number of packets of cereal sold per day is more than 124.5, when in fact the mean number is 124.5.

7. (a) Unbiased estimate of population mean, $\mu = \overline{x} = 10.2$.
Unbiased estimate of population variance, $\sigma^2 = s^2 = 0.0317$.

(b) p-value $= 0.0016038 < 0.05$. We reject H_0. There is sufficient evidence at the 5% significance level to conclude that the mean amount of rice in each packet is no longer 10 kg.

(c) $\alpha = 0.160$.

8. (a) p-value $= 0.0128 > 0.01$. WE do not reject H_0. There is insufficient evidence to conclude that the mean tail lengths of Proboscis Monkeys on the island is greater than 65 cm, thus Elsa's claim is valid at 1% level of significance.

(b) $\{t \in \mathbb{R} : t \le 60.7 \text{ or } t \ge 65.3\}$.
It means that there is a 0.05 probability that the test will conclude that the mean tail length of the Proboscis Monkeys is not 63 cm when in fact the mean tail length of the monkeys is 63 cm.