

Differentiation Techniques – Exercise 4

(a) Techniques of Differentiation – Exponential Function

1. Differentiate the following with respect to x .

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|---------------------------------|--|
| (a) e^{2x+3} | (b) e^{2-7x} |
| (c) $3e^{3-5x} + 4x^2$ | (d) $\frac{3}{e^x}$ |
| (e) $e^{2x} + \frac{1}{e^{2x}}$ | (f) $\frac{1}{6}e^{\frac{1}{2}x} - 2e^{-3x}$ |
| (g) $e^{\sqrt{x}}$ | (h) $\frac{1}{e^{\sqrt{2x}}}$ |

2. Differentiate the following with respect to x .

- | | |
|--|------------------------------|
| (a) $(e^{2x} + 3)^3$ | (b) $e^{2x+3} \sin 2x$ |
| (c) $\sqrt{2 - e^{3x}}$ | (d) e^{x^2+3x} |
| (e) $\frac{e^{\frac{1}{2}x}}{\cos 4x}$ | (f) $\frac{x + e^{2x}}{e^x}$ |
| (g) $e^{3-\sin 5x}$ | (h) $x^3 e^{6x}$ |

Notes:

- (a) $\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$
 (b) $\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$

Examples:

- (a) $\frac{d}{dx}(e^{5x-2}) = 5e^{5x-2}$
 (b) $\frac{d}{dx}(e^{x^2+4x}) = (2x + 4)e^{x^2+4x}$
 (c) $\frac{d}{dx}(e^{\cos x}) = (-\sin x)e^{\cos x}$

(b) Techniques of Differentiation – Logarithmic Function

3. Differentiate the following with respect to x .

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|--------------------------|--------------------------------|
| (a) $\ln(5x + 3)$ | (b) $\ln(4x + 2)^6$ |
| (c) $\ln(2x - 3)(x + 5)$ | (d) $\ln(\sin 3x - 3 \cos 4x)$ |
| (e) $\ln(x + e^x)$ | (f) $\ln(\sqrt{x^2 - 1} - x)$ |
| (g) $\frac{\ln x}{x}$ | (h) $x \ln x$ |

4. Differentiate the following with respect to x .

- | | |
|---|-----------------------------------|
| (a) $\ln\left(\frac{2x-1}{1-5x}\right)$ | (b) $\ln\sqrt{\frac{4x+1}{1-2x}}$ |
| (c) $\ln(\sec x)$ | (d) $\ln(x \sin 2x)$ |
| (e) $x^3 \ln(\cos^2 x)$ | (f) $\ln(e^{\tan 2x} + x)$ |
| (g) $\sqrt{1 + 3 \ln 5x}$ | |

Notes:

- (a) $\frac{d}{dx}(\ln(ax + b)) = \frac{a}{ax+b}$
 (b) $\frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)}$

Examples:

- (a) $\frac{d}{dx}(\ln(5x + 2)) = \frac{5}{5x+2}$
 (b) $\frac{d}{dx}(\ln\sqrt{3x+1}) = \frac{d}{dx}\left[\frac{1}{2}\ln(3x+1)\right] = \frac{1}{2}\left(\frac{3}{3x+1}\right)$
 (c) $\frac{d}{dx}\ln(2x+1)(4x-1) = \frac{d}{dx}[\ln(2x+1) + \ln(4x-1)] = \frac{2}{2x+1} + \frac{4}{4x-1}$
 (d) $\frac{d}{dx}\ln\left(\frac{2x+1}{1-3x}\right) = \frac{d}{dx}[\ln(2x+1) - \ln(1-3x)] = \frac{2}{2x+1} - \left(\frac{-3}{1-3x}\right)$

$\ln x^r = r \ln x$

• $\ln(xy) = \ln x + \ln y$

• $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

Miscellaneous Exercise

1. Differentiate the following with respect to x ,

(i) $\ln(x^2 + 5)^4$ (ii) $\ln\left(\frac{4}{1-3x^2}\right)$

(iii) $\ln[(x^2 - 1)\sqrt{x + 1}]$ (iv) $\ln(\sin 3x)$

(v) $e^{4x} \sin(2x + 1)$ (vi) $x^2 e^{x^2}$

(vii) $e^{-\cos x}$ (viii) $x^3 \ln(\sin^3 x)$

[(i) $\frac{8x}{x^2+5}$; (ii) $\frac{6x}{1-3x^2}$; (iii) $\frac{5x-1}{2(x-1)(x+1)}$; (iv) $3 \cot 3x$;

(v) $2e^{4x}[\cos(2x + 1) + 2 \sin(2x + 1)]$; (vi) $2xe^{x^2}(x^2 + 1)$; (vii) $\sin x e^{-\cos x}$;

(viii) $3x^2(x \cot x + \ln \sin^3 x)$

2. Differentiate the following with respect to x .

(a) $\ln\left(\frac{2x+3}{\sqrt{x}}\right)$ (b) $x(x^2 - 1)\sin \frac{x}{2}$

3. Differentiate the following with respect to x .

(a) $(4 - 3x^2)^5$ (b) $\ln \sqrt{\frac{6x}{2x-3}}$

4. Differentiate the following with respect to x .

(a) $\frac{2x}{\sqrt{2x+1}}$ (b) $\frac{e^{\sqrt{x}}}{\sec^2 x}$

5. Differentiate the following with respect to x .

(a) $\ln\left(\frac{x+2}{x-2}\right)$ (b) $\frac{e^{\sin 2x}}{x+1}$

Answers:

2. (a) $\frac{2}{2x+3} - \frac{1}{2x}$ (b) $(3x^2 - 1)\sin \frac{x}{2} + \frac{1}{2}(x^3 - x)\cos \frac{x}{2}$

3. (a) $-30x(4 - 3x^2)^4$ (b) $\frac{3}{2x(3-2x)}$

4. (a) $\frac{2(1+x)}{(2x+1)^{\frac{3}{2}}}$ (b) $\frac{e^{\sqrt{x}}}{2\sqrt{x}} \cos^2 x - 2e^{\sqrt{x}} \cos x \sin x$

5. (a) $\frac{-4}{(x+2)(x-2)}$ (b) $\frac{[2(x+1)\cos 2x - 1]e^{\sin 2x}}{(x+1)^2}$

6. Given that $y = 4e^{-2x} - 1$, find

(i) the value of y when $x = 0$

(ii) the value of $\frac{dy}{dx}$ when $\frac{4}{e^{2x}} = 5$.

Sketch the graph of $y = 4e^{-2x} - 1$.

[(i) 3, (ii) -10]

7. Given that $y = e^x \cos x$, express $\frac{dy}{dx}$ in the form $ke^x \cos(x + \alpha)$, stating clearly suitable values for k and α .

$[\sqrt{2}; \frac{\pi}{4}]$

8. Given that $y = x^2 e^{3x}$, write down an expression for $\frac{dy}{dx}$ and hence determine the values of x for which y is stationary.

$[xe^{3x}(2 + 3x); 0 \text{ or } -\frac{2}{3}]$

9. **O-Level November 2001**

Given that $y = Ae^{kx}$, where A and k are constants, find an expression for $\frac{dy}{dx}$. Hence find the value of k and of A for which $\frac{dy}{dx} - 3y = 4e^{2x}$.

$[k = 2; A = -4]$

10. **St-nicks Prelim November 2008**

Express $\frac{d}{dx} [\ln \cot 4x]$ in the form $\frac{k}{\sin px}$, where k and p are constants.

$[\frac{-8}{\sin 8x}]$

11. **Xinmin Secondary School SA2 2010**

Given that $y = \ln(x^2 e^{3x-2})$, where $x > 0$, show that

$$\frac{1}{x} \frac{dy}{dx} + \frac{d^2y}{dx^2} = \frac{3}{x}$$