Maxima & Minima – Revision 1

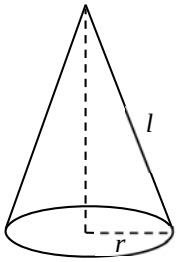
Ang Mo Kio Secondary School Prelim Exam 2008

1. A piece of wire of length 8cm is cut into two pieces, one of length x cm, the other of length $(8 - x)$ cm. The piece of length x cm is bent to form a circle of radius r cm. The other pieces is bent to form a square.
   (a) Express r in terms of x.
   (b) Show that the sum of area A cm^3 enclosed by these two pieces of wire is given by $A = \left(\frac{1}{4\pi} + \frac{1}{16}\right)x^2 - x + 4$.
   (c) Given that x varies, find the value of x for which A has a stationary value.
   (d) Find the corresponding value of A and determine whether it is a maximum or minimum value..

[(a) $r = \frac{x}{2\pi}$ (c) 3.52 (d) 2.24cm^2, minimum]

Catholic High School Prelim Exam 2008

2. The diagram shows a roof in the shape of a right inverted circular cone whose radius is r m and its slanted height l m. The sloping surface of the roof is covered with a sheet of thin metal whose area is $4\sqrt{3}\pi$ m^2.
   (a) Express l in terms of r and show that the volume of the cone, V cm^3 is given by $V = \frac{\pi}{3}r\sqrt{48 - r^4}$.
   (b) Given that r can vary, find
      (i) an expression for $\frac{dy}{dx}$,
      (ii) the value of r for which the V has a stationary value.



[(b)(ii) $r = 2$]

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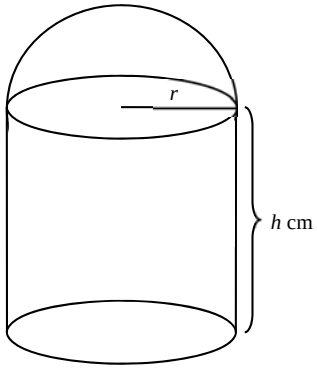
3. The volume of a container in the shape of an open right circular cylinder of radius r cm and height h cm is 500π cm^3.
   (a) Express h in terms of r.
   (b) A hemispherical lid is attached to the container as shown in the diagram below. External surfaces of the container and the lid are painted.



It costs 3 cents per cm^2 to paint the cylindrical surface and 4 cents per cm^2 to paint the base and the lid.
Let \$$C$ be the total cost of painting the container.
Show that $C = 3\pi\left(\frac{r^2}{25} + \frac{10}{r}\right)$.

   (c) Find the value of r which gives the minimum value of C and find the minimum cost of painting the container and the lid, giving your answer to the nearest cent.

[(a) $h = \frac{500}{r^2}$ (b) $3\pi\left(\frac{r^2}{25} + \frac{10}{r}\right)$ (c) r = 5cm, min value of C = \$28.27]

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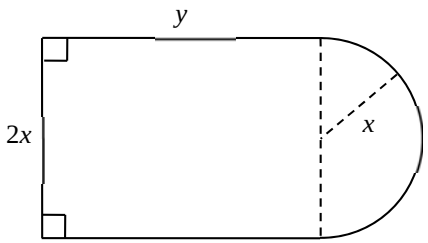
4.



A piece of wire of length 300 cm is bent to form an enclosure consisting of a rectangle of length y cm and width $2x$ cm with a semicircle of diameter $2x$ cm as shown in the figure.

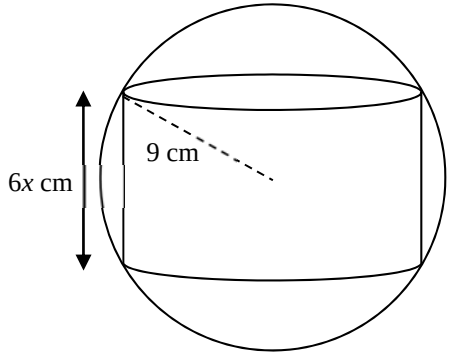
(a) Show that the area A cm^2, of the enclosure is given by $A = 300x - \frac{\pi + 4}{2}x^2$.

(b) Find the value of x for which there is a stationary value for A and determine whether it is a maximum or a minimum.

[x = 42.0 cm, A is maximum]

Tanjong Katong Secondary School Prelim Exam 2008

5. A solid right cylinder is removed from a solid sphere of radius 9 cm as shown. The height of the cylinder is $6x$ cm.



(i) Show that the volume, V cm^3, of the cylinder is given by $V = 486\pi x - 54\pi x^3$.

Given that x can vary,

(ii) find the value of x for which V has a stationary value and determine whether this value of V is a maximum or a minimum.

[(a)(ii) $x = \frac{\pi}{3}$ (b)(ii)$x = \sqrt{3}$, V is maximum]

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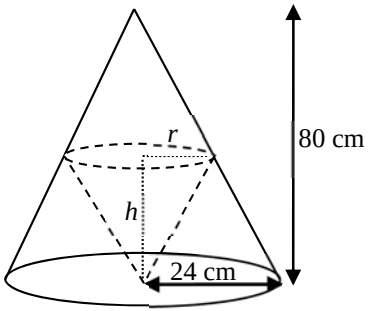
6.



The diagram shows a solid cone with base radius 24 cm and height 80 cm. A cone of radius r cm and height h cm is to be removed.

(a) Express h in terms of r.

(b) Hence show that the volume, V cm^3, of the cone to be removed is given by $V = \frac{80\pi}{3}r^2 - \frac{10\pi}{9}r^3$.

(c) Calculate the value of r for which V has a stationary value. Hence, find the stationary value of V and determine whether it is a maximum or minimum value.

[(a) $h = 80 - \frac{10}{3}r$ (c) 7150]

Bukit Panjang Govt High Prelim Exam 2008

7. ΔABC is an isosceles triangle with $AB = AC = 10$ cm and $BC = 12$ cm. A rectangle $PQRS$ is drawn inside the triangle with PQ on BC, and S and R on AB and AC respectively.

(a) If $PQ = x$ cm, show that the area, A cm^2 of the rectangle is given by $A = \frac{2x}{3}(12 - x)$.

(b) Hence, or otherwise, find the maximum area of the rectangle.

(c) If $AN = y$ cm, where N is the mid-point of SR, express y in terms of x.

(d) Given that x is changing at the rate of $\frac{3}{1+x}$ cm/s, find the rate at which y is changing when $A = 24$ cm^2.