

Binomial Distribution

1. Over a long period of time, it is found that 20% of candidates who take a particular piano examination fail the examination.
 - (i) Find the probability that, in a group of 10 randomly chosen candidates who take the examination, exactly 2 will fail. [2]
 - (ii) It is given that 15% of the candidates who **pass** the piano examination are awarded a distinction. Find the probability that, in a randomly chosen group of 10 candidates who take the examination, fewer than 2 will be awarded a distinction. [3]
 - * (iii) ~~Use a suitable approximation to estimate~~ ^{Find} the probability that, in a group of 50 randomly chosen candidates who take the examination, at most 12 will fail. You should state the mean and variance of the distribution used ~~in the approximation.~~ [4]

(2009/P1/Q10)

2. The probability of any sunflower seed germinating when it is sown is 0.7, independently of all other sunflower seeds. Find the probability that, when 8 seeds are sown,
 - (i) exactly 6 will germinate, [2]
 - (ii) at least 6 will germinate. [2]

Find

* 60 sunflower seeds are sown. ~~Use a suitable approximation to estimate~~ the probability that fewer than 40 will germinate. You should state the mean and variance ~~of the approximation.~~ [4]

(2010/P1/Q9)

3. Jon attempts a puzzle in his daily newspaper each day. The probability that he will complete the puzzle on any given day is 0.8, independently of any other day.
 - (i) Find the probability that, in a given week of 7 days, Jon will complete the puzzle
 - (a) exactly 3 times, [1]
 - (b) at least 5 times. [2]
 - (ii) Find the probability that, over a period of 10 weeks, Jon completes the puzzle at least 5 times each week. [2]
 - * (iii) ~~Using a suitable approximation,~~ find the probability that, over a period of 10 weeks, Jon completes the puzzle at least 50 times in total. State the mean and variance ~~of the approximation.~~ [4]

(2011/P1/Q10)

4. 'Sunbrite' plants are sold in trays of 12 plants. For any Sunbrite plant, the probability that it flowers is 0.8, independently of all other Sunbrite plants. Find the probability that from one tray of Sunbrite plants

- (i) exactly 10 will flower, [2]
(ii) fewer than 8 will flower. [2]

A gardener A buys 8 trays of Sunbrite plants.

- * (iii) ~~Use a suitable approximation to estimate the probability that more than 75 plants will flower. State the mean and variance of the distribution that you use.~~ ^{Find} [4]

Two other gardeners, B and C, each buy 8 trays of Sunbrite plants.

- * (iv) Find the probability that, for at least two of the three gardeners A, B and C, more than 75 of their plants will flower. [3]

(2012/P1/Q10)

5. A shop sells batteries in packs of 10. An advertiser claims that individual batteries each have a lifetime of at least 100 hours. The probability that an individual battery has a lifetime less than 100 hours is 0.2, independently of all other batteries.

- (i) Find the probability that, in a randomly chosen pack of 10 batteries, each of the batteries satisfies the advertiser's claim. [1]

Customers are satisfied if at least 8 of the batteries in a pack have a lifetime of at least 100 hours.

- (ii) Find the probability that a randomly chosen pack will satisfy customers. [3]

A customer buys a batch of 80 packs of these batteries.

- * (iii) ~~Using a suitable approximation, estimate the probability that at least 75% of packs in the batch will satisfy the customer. State the mean and variance of the distribution that you use.~~ ^{Find} [4]

(2013/P1/Q8)

6. A bakery produces two kinds of cake. One kind of cake contains fruit, and the other kind contains no fruit. There is a constant probability that a cake contains fruit. The cakes are sold in packs of 6. Each pack has a random selection of cakes. For these packs, the mean number of cakes containing fruit is 2.4.

- (i) Find the probability that a pack chosen at random has
(a) no cakes containing fruit, [2]
(b) at most two cakes containing fruit. [1]

A customer buys 8 packs of cakes for a party.

- (ii) Find the probability that at least 4 of these packs have at most two cakes containing fruit. [3]

A supermarket stocks 150 of these packs of cakes.

- ~~Find~~
* (iii) ~~Using a suitable approximation, estimate~~ the probability that more than half of the packs have at most two cakes containing fruit. You should state the mean and variance of any distribution that you use. [4]
(2014/P1/Q9)

7. Kai throws a fair die 8 times. Find the probability that he obtains a six

- (i) exactly three times, [1]
(ii) fewer than four times. [2]

Lam throws a fair die 600 times.

- ~~Find~~
* (iii) ~~Using a suitable approximation, estimate~~ the probability that the number of times he obtains a six is between 90 and 100 inclusive. State the mean and variance of the distribution that you use. [4]
(2015/P1/Q9)

8. Watch batteries are supplied to a shop in packs of 8. The probability that any randomly chosen battery has a lifetime of less than two years is 0.6, independently of all other batteries.

- (i) For a single pack of batteries, find the probability that
(a) all of the batteries have a lifetime of less than two years, [1]
(b) at least half of the batteries have a lifetime of less than two years. [2]
(ii) For any 4 packs of batteries, find the probability that, for no more than 2 of the packs, at least half of the batteries have a lifetime of less than two years. [2]

- ~~Find~~
(iii) A customer buys 10 packs of these batteries. ~~Use a suitable approximation to estimate~~ the probability that at least 40 of these batteries have a lifetime of less than two years. State the mean and variance of the distribution that you use. [4]
(2016/P1/Q9)

9. Printers in a busy office produce large numbers of documents each week. The ink cartridges used in the printers often need replacing. The probability that an ink cartridge will last for one week or more is 0.7, independently of all other cartridges. The cartridges are supplied in boxes of 8. A box is selected at random.

- (i) Find the probability that exactly 5 of the cartridges in the box will last for one week or more. [1]
(ii) Find the probability that at least half of the cartridges will last for less than one week. [2]

The office has 6 boxes of ink cartridges in stock.

- (iii) Find the probability that, for at most 2 of the boxes, at least half of the cartridges will last less than one week. [2]
(2017/P1/Q7)