Curve Sketching: some discussion questions

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Last updated April 10, 2020.
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1 Rational Functions

1.1 Practice on basic techniques

Sketch the following curves, showing clearly the asymptotes and the coordinates of points of intersection with the axes as well as any stationary points (if applicable)

1. $y = \dfrac{5}{x+3}$.
2. $y = \dfrac{2}{3-2x}$.
3. $y = \dfrac{2x+1}{x-1}$.
4. $y = \dfrac{x}{3x-1}$.
5. $y = \dfrac{2x^2+3x+1}{x}$.
6. $y = \dfrac{x^2+3x+1}{x+2}$.

1.2 Use of the discriminant

7. Using the curve sketched in question 5, $y = \dfrac{2x^2+3x+1}{x}$, find the set of values that y can take, leaving your answer to 2 decimal places.
8. Without the use of a graphing calculator, use an algebraic method to find the set of values that y can take for $y = \dfrac{2x^2+3x+1}{x}$.

1.3 Questions with some "tricks"

Sketch the following curves, showing clearly the asymptotes and the coordinates of points of intersection with the axes as well as any stationary points (if applicable).

9. $y = \dfrac{2x^2 - x + 2}{2x - 4}$.

10. $y = \dfrac{2x^2 + 4x - 1}{x - 1}$.

11. $y = \dfrac{3(1-x)}{(3x-2)(x+2)}$.

12. $y = \dfrac{x^2 - 18x + 81}{x^2 - 9}$.

1.4 Some more advanced follow up questions

13. (Follow up on question 11)
    How many roots does the equation

    $$\frac{3(1-x)}{(3x-2)(x+2)} = -3x - 1$$

    have?

14. (Follow up on question 12)
    Find the range of values of k such that the equation

    $$\frac{x^2 - 18x + 81}{x^2 - 9} = k$$

    has two real and distinct solutions.

15. (Follow up on question 5)
    Find the set of values k can take such that the equation

    $$\frac{2x^2 + 3x + 1}{x} = kx + 3$$

    has no solutions.

2 Parametric equations

2.1 Basic curve sketching techniques

Sketch the following curves, showing clearly the asymptotes (if applicable) and the coordinates of points of intersection with the axes.

1. $x = 3\cos t + 2, \quad y = \sin t, \quad 0 \le t \le 2\pi.$
2. $x = 3\cos t + 2, \quad y = \sin t, \quad 0 \le t \le \frac{\pi}{2}.$
3. $x = t^3 - 1, \quad y = \dfrac{1}{t-2}, \quad t \in \mathbb{R}, t \neq 2.$

2.2 Conversion to Cartesian equation

4. Convert the equation of the curve in question 3 to cartesian form.
5. Convert the equation of the curve in question 1 to cartesian form.

3 Conics

3.1 Basic techniques, ellipses, hyperbolas

Sketch the following curves, showing clearly the asymptotes and the coordinates of any stationary points (if applicable).

1. $\dfrac{(x-1)^2}{2^2} + \dfrac{(y+2)^2)}{9} = 1.$
2. $\dfrac{(x-1)^2}{2^2} - \dfrac{(y+2)^2)}{9} = 1.$
3. $\dfrac{(y+2)^2)}{9} - \dfrac{(x-1)^2}{2^2} = 1.$

3.2 Completing the square to obtain conics

Sketch the following curves, showing clearly the asymptotes and the coordinates of any stationary points (if applicable).

1. $x^2 + 2x + y^2 - 6y - 6 = 0.$
2. $x^2 - 2x - y^2 = 0.$