DRV Example

The pdf for X (both X_1 and X_2 has the same distribution):

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Now $W = |X_1 - X_2|$. What is the meaning of $|X_1 - X_2|$? It means that we take a random number for x_1, for example, $x_1 = 3$, and a randomly number for x_2, for example, $x_2 = 5$. Then $w = |x_1 - x_2| = 2$. This is associated with a certain probability $\left(\frac{1}{36}\right.$ for this example$\left.\right)$. To work out the probabilities for W, a table of outcomes is useful.

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

This gives us the following pdf for W:

w	0	1	2	3	4	5
$P(W=w)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

Now try finding $E(W)$.

Next we have $Q = X_1 - X_2$. This is similar to W, except that, without a modulus, Q can take negative numbers. Let's try to find the pdf for Q:

q	-5	-4	…	4	5
$P(Q=q)$	$\frac{1}{36}$	$\frac{2}{36}$	…	$\frac{2}{36}$	$\frac{1}{36}$

Let's find $E(Q^2), E(W^2), Var(Q)$ and $Var(W)$ to answer the questions.

Answers

(ii) $E(W) = \frac{35}{18}$.

(iii) $E(W^2) = \frac{35}{6} = E(Q^2)$.

(iv) Since $E(W) = \frac{35}{18}$ and $E(Q) = 0$, $Var(W) = \frac{665}{324} \neq \frac{35}{6} = Var(Q)$.