2017 H2 MA Prelim Compilation - DRV (18 Questions)

ACJC Prelim 9758/2017/02/Q6

Alex and his friend stand randomly in a queue with 3 other people. The random variable X is the number of people standing between Alex and his friend.

(i) Show that $\text{P}(X = 2) = 0.2$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $\text{E}(X)$ and $\text{E}(X - 1)^2$. Hence find $\text{Var}(X)$. [3]

Answers

(ii) P(X=0)=0.4, P(X=1)=0.3, P(X=2)=0.2, P(X=3)=0.1;

(iii) 1, 1, 1.

CJC Prelim 9758/2017/02/Q10

A box contains 2 red balls, 3 green balls and x blue balls, where $x \in \mathbb{Z}, x \geq 5$. A game is played where the contestant picks 5 balls from the box without replacement. The total score, S, for the contestant is the sum of the number of green balls chosen and thrice the number of red balls chosen. The blue balls will not contribute any points, unless all 5 balls are blue. If all the 5 balls are blue, the score will be 25 points.

(i) Show that $\text{P}(S = 6) = \dfrac{20x(x^2 - 3x + 14)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$. [2]

(ii) Given that $\text{P}(S = 6) = \dfrac{5}{63}$, calculate x. [2]

(iii) Complete the probability distribution table for S. [4]

s	1	2	3	4	5	6	7	8	9	25
$\text{P}(S = s)$		$\frac{5}{42}$	$\frac{5}{63}$		$\frac{5}{21}$	$\frac{5}{63}$		$\frac{5}{84}$	$\frac{1}{252}$	

(iv) Evaluate $\text{E}(S)$ and find the probability that S is more than $\text{E}(S)$. [2]

(v) Find the probability that there are no green balls drawn given that S is more than $\text{E}(S)$. [2]

Answers

(ii) $x = 5$

(iv) $\dfrac{127}{252}$ or 0.504

(v) $\frac{11}{127}$

DHS Prelim 9758/2017/02/Q5

A new game has been designed for a particular casino using two fair die. In each round of the game, a player places a bet of \$2 before proceeding to roll the two die. The player's score is the sum of the results from both die. For the scores in the following table, the player keeps his bet and receives a payout as indicated.

Score	Payout
9 or 10	\$1
2 or 4	\$5
11	\$8

For any other scores, the player loses his bet.

Let X be the random variable denoting the winnings of the casino from each round of the game.

(i) Show that $\mathrm{E}(X)=\frac{1}{12}$ and find $\mathrm{Var}(X)$. [4]

(ii) $\bar{X}$ is the mean winnings of the casino from n rounds of this game. Find $\mathrm{P}(\bar{X}>0)$ when $n=30$ and $n=50\,000$. Make a comparison of these probabilities and comment in context of the question. [3]

Answers

(i) $\frac{1307}{144}$ or 9.08 (to 3sf)

(ii) For n = 30, $\mathrm{P}\left(\bar{X}>0\right)=0.560$ (to 3sf)

For n = 50000, $\mathrm{P}\left(\bar{X}>0\right)=1.00$ (to 3sf)

It is almost certain that casino will win in the long run.

HCI Prelim 9758/2017/02/Q6

A biased tetrahedral (4-sided) die has its faces numbered '–1', '0' ,'2' and '3'. It is thrown onto a table and the random variable X denotes the number on the face in contact with the table. The probability distribution of X is as shown.

x	-1	0	2	3
$\mathrm{P}(X=x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$

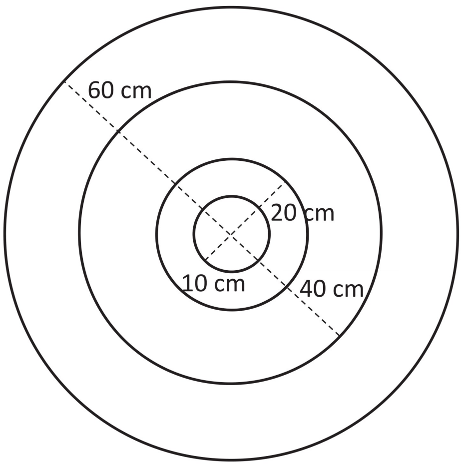
(i) The random variable Y is defined by X_1+X_2, where X_1 and X_2 are 2 independent observations of X. Show that $P(Y=2)=\frac{3}{16}$. [2]

(ii) In a game, a player pays \$2 to throw two such biased tetrahedral dice simultaneously on a table. For each die, the number on the face in contact with the table is the score of the die. The player receives \$16 if the maximum of the two scores is -1, and receives \$3 if the sum of the two scores is prime. For all other cases, the player receives nothing. Find the player's expected gain in the game. [4]

Answers
(ii) –\$0.25

IJC Prelim 9758/2017/02/Q7

An archer shoots an arrow into a circular target board that has a radius of 60 cm. The target board further consists of three inner concentric circular sections, with radii 40 cm, 20 cm and 10 cm respectively as shown in the diagram.



The archer scores

- 50 points if the arrow lands in the centre circle of radius 10 cm,
- 20 points if the arrow lands in the ring with outer radius 20 cm,
- 10 points if the arrow lands in the ring with outer radius 40 cm,
- 0 point otherwise.

Assume that the arrow will definitely hit the target board and is equally likely to hit any portion of the target board.

(i) Let X be the number of points scored for one arrow shot. Find the expectation of X, leaving your answer in 4 significant figures. [3]

(ii) Interpret, in this context, the value obtained in part **(i)**. [1]

(iii) The archer shot at the target board forty times. Find the probability that the average score obtained by the archer is between 10 and 20 points (inclusive). [4]

Answers
(i) 6.389

(iii) 0.00965

JJC Prelim 9758/2017/02/Q5

The probability distribution of a discrete random variable, X, is shown below.

x	1	2
$\mathrm{P}(X=x)$	a	b

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ in terms of a. [5]

Answers

$\mathrm{E}(X) = 2-a$ and $\mathrm{Var}(X) = a-a^2$

MJC Prelim 9758/2017/02/Q6

The probability function of X is given by

$$\mathrm{P}(X=x)=\begin{cases}(2x-1)\theta & \text{if } x=1,2,3\\ k & \text{if } x=4\\ 0 & \text{otherwise}\end{cases}$$

where $0<\theta<\frac{1}{9}$.

(i) Show that $k=1-9\theta$. Find, in terms of θ, the probability distribution of X. [2]

(ii) Find $\mathrm{E}(X)$ in terms of θ and hence show that $\mathrm{Var}(X)=26\theta-196\theta^2$. [3]

(iii) The random variable Y is related to X by the formula $Y=a+bX$, where a and b are non-zero constants. Given that $\mathrm{Var}(Y)=\frac{1}{3}b^2$, find the value of θ. [3]

Answers

(i)

x	1	2	3	4
$\mathrm{P}(X=x)$	θ	3θ	5θ	$1-9\theta$

(ii) $\mathrm{E}(X)=4-14\theta$

(iii) $\theta=0.0144$

	NJC Prelim 9758/2017/02/Q7
	There are three identically shaped balls, numbered from 1 to 3, in a bag. Balls are drawn one by one at random and with replacement. The random variable X is the number of draws needed for any ball to be drawn a second time. The two draws of the same ball do not need to be consecutive. **(i)** Show that $\mathrm{P}(X=4)=\frac{2}{9}$ and find the probability distribution of X. [3] **(ii)** Show that $\mathrm{E}(X)=\frac{26}{9}$ and find the exact value of $\mathrm{Var}(X)$. [3] **(iii)** The mean for forty-four independent observations of X is denoted by $\overline{X}$. Using a suitable approximation, find the probability that $\overline{X}$ exceeds 3. [3]
	Answers (i) $\mathrm{P}(X=4)=\frac{2}{9}$ (ii) $\mathrm{E}(X)=\frac{26}{9}$ (iii) $P(\overline{X}>3)=0.159$

	NYJC Prelim 9758/2017/02/Q5
	From past records, the number of days of hospitalization for an individual with minor ailment can be modelled by a discrete random variable with probability density function given by $$P(X=x)=\begin{cases}\dfrac{6-x}{15}, & \text{for } x=1,2,3,4,5,\\ 0, & \text{otherwise.}\end{cases}$$ An insurance policy pays \$100 per day for up to 3 days of hospitalization and \$25 per day of hospitalization thereafter. **(i)** Calculate the expected payment for hospitalization for an individual under this policy. [4] **(ii)** The insurance company will incur a loss if the total payout for 100 hospitalisation claims under this policy exceed \$24000. Using a suitable approximation, estimate the probability that the insurance company will incur a loss for 100 such claims. [4]
	Answers

	(i) $213\frac{1}{3}$ (ii) 0.00217

PJC Prelim 9758/2017/02/Q6

An unbiased disc has a single dot marked on one side and two dots marked on the other side. A tetrahedral die has faces marked with score of 1, 2, 3, and 4. The probability of getting a score of 1, 2, 3, and 4 is $\frac{1}{5}, p, \frac{1}{5}$ and q respectively, where $p, q \in [0,1]$.

A game is played by throwing the disc and the die together. The random variable S is the sum of the score showing on the die and twice the number of dots showing on the disc.

(i) Find $\text{P}(S=6)$. [2]

Given that $\text{P}(S=4)=\frac{1}{6}$,

(ii) calculate the values of p and q, [2]

(iii) and find the probability distribution of S. [2]

Answers

(i) $\frac{3}{10}$

(ii) $p=\frac{1}{3}$, $q=\frac{4}{15}$

(iii)

s	3	4	5	6	7	8
$\text{P}(S=s)$	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{2}{15}$

RI Prelim 9758/2017/02/Q7

An unbiased cubical die has the number 1 on one face, the number 2 on two faces and the number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the die twice. Adrian pays Benny $\$a$ if the total score is 2 and \$3 if the total score is 3. However, if the total score is 4, Benny pays Adrian \$2. No payment is made otherwise.

(i) Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4]

The random variable X represents Benny's winnings in each round.

(ii) Given that $a = 6$, find the probability distribution of X. Hence, help Benny decide if he should accept Adrian's invitation to play the game. Justify your answer. [5]

(iii) Determine the value of a for the game to be fair. [1]

Answers

(i) 0.134 (ii) $\frac{5}{18}$ (iii) $a = 8$

RVHS Prelim 9758/2017/02/Q7

A board game simulates players attacking each other by throwing tetrahedral (8-sided) dice. When attacking, the player throws an attack die once. An attack die has 5 of the sides printed with the number "0", 2 of the sides printed with the number "1", and 1 of the sides printed with the number "2". After the attacking player has thrown the attack die, the defending player throws a defence die once. A defence die has 2 of the sides printed with the number "0", 4 of the sides printed with the number "1" and 2 of the sides printed with the number "2". The damage dealt during a round is equal to the score shown on the attack die minus the score shown on the defence die. If the score on the defence die is more than the score on the attack die, the damage dealt will be zero.

Let A denotes the score on an attack die, and D denotes the score on a defence die.

(i) Write down the probability distributions for A and D. Hence find the expected value and variance of $A - D$. [4]

Let X denote the damage dealt during a round.

(ii) Find the probability distribution for X. Hence find the expected value and variance of X. [5]

(iii) Explain why, in the context of the question, $\text{E}(X) > 0$ when $\text{E}(A) < \text{E}(D)$. [1]

Answers

(i) $\frac{-1}{2}$, 1

(ii) $\frac{3}{16}$, $\frac{55}{256}$

	SAJC Prelim 9758/2017/02/Q8
	A fairground game involves trying to hit a moving target with a gunshot. A round consists of a **maximum** of 3 shots. Ten points are scored if a player hits the target. The **round** ends **immediately** if the player misses a shot. The probability that Linda hits the target in a single shot is 0.6. All shots taken are independent of one another. **(i)** Find the probability that Linda scores 30 points in a round. [2] The random variable X is the number of points Linda scores in a round. **(ii)** Find the probability distribution of X. [3] **(iii)** Find the mean and variance of X. [4] **(iv)** A game consists of 2 rounds. Find the probability that Linda scores more points in round 2 than in round 1. [2]
	Answers i) 0.216 iii) 11.76, 137.7024 iv) 0.358

	SRJC Prelim 9758/2017/02/Q5
	A random variable X has the probability distribution given in the following table.

x	2	3	4	5
$\mathrm{P}(X=x)$	0.2	a	b	0.45

Given that $E\left(\left|X-4\right|\right)=\dfrac{11}{10}$, find the values of a and b. [3]

Two independent observations of X are taken. Find the probability that one of them is 2 and the other is at most 4. [2]

Answers

$a=0.25$ and $b=0.1$, 0.18

	TJC Prelim 9758/2017/02/Q6
	In a game at the carnival, a player rolls discs onto a board containing squares, each of which bears one of the numbers 1, 2, 5 or 10. If a disc does not land within a square, the player receives nothing. The probability that the disc does not land within the square is $\frac{3}{4}$. If a disc lands within a square, the player receives the same amount (in dollars) as the number in the square. Given that a disc falls within a square, the probabilities of landing within a square with

the numbers 1, 2, 5 and 10 are 0.5, 0.3, 0.12 and 0.08 respectively. It is assumed that the rolls of the discs are independent.

(i) A player pays \$5 to play the game and is given n discs. Find n if the game is fair. [4]

(ii) If a player is allowed to roll 3 discs for \$2, find the probability that the player will have a profit of \$10. [4]

(i) Answers
$n = 8$
0.0789

TPJC Prelim 9758/2017/02/Q5

An unbiased six-sided die is rolled twice. The random variable X represents the higher of the two values if they are different, and their common value if they are the same. The probability distribution of X is given by the formula

$$P(X = r) = k(2r - 1) \quad \text{for } r = 1,\ 2,\ 3,\ 4,\ 5,\ 6.$$

(i) Find the exact value of k, giving your answer as a fraction in its simplest form. [2]

(ii) Find the expectation of X. [2]

A round of the game consists of rolling the unbiased six-sided die twice, and X is taken as the score for the round. A player plays three rounds of the game.

(iii) Find the probability that the total score for the three rounds is 16. [2]

Answers
(i) $\dfrac{1}{36}$
(ii) $\dfrac{161}{36}$
(iii) 0.112

VJC Prelim 9758/2017/02/Q7

Four digits are randomly selected from the set $\{1,2,3,4,5,6,7,8,9\}$ to form a four-digit number. Repetitions are not allowed.

(i) Find the probability that none of the digits in the four-digit number are odd. [2]

The random variable X denotes the number of odd digits in the four-digit number formed.

(ii)	Show that $P(X=1)=\frac{10}{63}$, and find the rest of the probability distribution of X, giving each probability as a fraction in its lowest terms. [3]
(iii)	Find the expectation and variance of X. [3]
(iv)	Two independent observations of X are denoted by X_1 and X_2. Find $P(\lvert X_1 - X_2 \rvert < 3)$. [4]

Answers

(i) $\frac{1}{126}$

(ii)

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{126}$	$\frac{10}{63}$	$\frac{10}{21}$	$\frac{20}{63}$	$\frac{5}{126}$

(iii) $E(X)=\frac{20}{9}$, $\text{Var}(X)=\frac{50}{81}$

(iv) $\frac{7793}{7938}$

	YJC Prelim 9758/2017/02/Q7
	A game is played with a set of 4 cards, each distinctly numbered 1, 2, 3 and 4. A player randomly picks a pair of cards without replacement. If the sum of the cards' numbers is an odd number, the sum is the player's score. If the sum of the two cards' number is an even number, the player randomly picks a third card from the remaining cards. The square of the third card's number is the player's score.
	(i) Find the probability that a player obtains a score of 4. [2]
	(ii) Find the probability distribution of a player's score, S. Hence, find the expected score of a player. [4]
	(iii) Find the probability that a player obtains a score lower than 5, given that he draws three cards. [3]

Answers

(i) $\frac{1}{12}$

(ii)

s	1	3	4	5	7	9	16
$P(S = s)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{4}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$

Expected score = $\frac{35}{6}$

(iii) $\frac{1}{2}$