Differentiation applications: a summary

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5.1 Equations of tangents

- Step 1: differentiate given equation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.
- Step 2: figure out what the x and y coordinates of the point is.
- Step 3: find the gradient of the tangent, m, by substituting x into $\frac{\mathrm{d}y}{\mathrm{d}x}$.
- Step 4: Use the formula $\boxed{y - y_1 = m(x - x_1)}$ where the point is (x_1, y_1).
- Step 5: Rearrange to make y the subject.

5.2 Increasing/decreasing functions

- A curve is **increasing** if $\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} > 0}$.
- A curve is **decreasing** if $\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} < 0}$.

5.3 Stationary/maximum/minimum points

- Step 1: differentiate the given equation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.
- Step 2: at **stationary/maximum,minimum** points, $\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$.
- Step 3: solve for x.
- Step 4: answer the question (do they want x? Coordinates?)
- Step 5: for maximum/minimum points, we have to test for the **nature** (refer to next subsection).

5.3.1 Testing for nature of stationary points: Method 1 (first order test)

x	a^-	a	a^+
$\frac{dy}{dx}$	< 0	0	> 0
Shape	╲	—	╱

Minimum point

x	a^-	a	a^+
$\frac{dy}{dx}$	> 0	0	> 0
Shape	╱	—	╱

Stationary point of inflexion

x	a^-	a	a^+
$\frac{dy}{dx}$	> 0	0	< 0
Shape	╱	—	╲

Maximum

5.3.2 Testing for nature of stationary points: Method 2 (second order test)

- Differentiate one more time to get $\frac{d^2y}{dx^2}$.
- Sub in our value of x.
- If $\boxed{\dfrac{d^2y}{dx^2} > 0}$, we have a **minimum**.
- If $\boxed{\dfrac{d^2y}{dx^2} < 0}$, we have a **maximum**.
- If $\frac{d^2y}{dx^2} = 0$, no conclusion (we have to use the first order test instead).

5.4 Maxima/minima (problem sums)

- Step 1: To maximise/minimise V, for example, we find a formula for V in terms of just one variable (x for example).
- Step 2: Differentiate to find $\frac{dV}{dx}$.
- Step 3: $\boxed{\dfrac{dV}{dx} = 0}$.
- Step 4: Solve for x.
- Step 5: Answer the question (do we want x? V? A?)
- Step 6: Test that it is indeed a maximum/minimum.

5.5 Rates of change

- The rate of change of V is given by $\frac{dV}{dt}$.
- If $\frac{dV}{dt} > 0$, V is increasing. If $\frac{dV}{dt} < 0$, V is decreasing.
- Step 1: Find a formula for V in terms of just one variable (x for example).
- Step 2: Differentiate to find $\frac{dV}{dx}$.
- Step 3: Use the chain rule equation: $\boxed{\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}}$.
- Step 4: Sub in given values and answer the question.