

1. [SAJC 18 MYE (modified)]

An insurance agent tries to sell a health insurance policy to a randomly selected customer. On average, the probability that a customer will purchase the policy is 0.25. Every week, the insurance agent tries to sell the policy to n different customers. If the agent is not successful in selling the policy to a particular customer, he does not attempt to sell the policy to the same customer again. Each customer is only entitled to purchase the policy once.

- (a) An insurance agent is expected to achieve a mean of at least 3 sales per week. Find the least number of customers an insurance agent should contact per week in order to meet this requirement. [3]

A week is considered 'good' for the insurance agent if he manages to sell the policy to at least 4 customers during the week. It is given that the insurance agent contacts 20 randomly selected customers per week. Find

- (b) the probability that a randomly selected week is considered 'good', [2]
 (c) the probability that in a year of 52 working weeks, the insurance agent has between 24 to 36 'good' weeks inclusive. [3]

2. [IJC 18 MYE (modified)]

A large number of people attend the movie premiere of 'Revenger: Finite War' and each of them receives a free hamper. There are two types of hampers, either Type-A or Type-B. On average, 15% of the people receive Type-A hampers. A random sample of 10 people is chosen and the number of people who receive Type-A hampers is the random variable A .

- (a) Find the probability that there are less than 3 people who receive Type-A hampers. [1]
 (b) Given that at least 7 people receive Type-B hampers, find the probability that at least 2 people receive Type-A hampers. [2]
 (c) Find the modal number of people receiving Type-B hampers. [2]

Some of the hampers are considered 'lucky' hampers as they each contain an additional gift of movie vouchers. It is known that 70% of Type-A hampers are 'lucky' hampers, while 50% of Type-B hampers are 'lucky' hampers.

- (d) Given that Tony receives a 'lucky' hamper, find the probability that it is a Type-A hamper. [3]

3. [DHS 17 Prelims (modified)]

A sample of 5 people is chosen from a village of large population. The number of people in the sample who are underweight is denoted by X .

- (a) On average, the proportion of people in the village who are underweight is p . It is known that the mode of X is 2. Use this information to show that $\frac{1}{3} < p < \frac{1}{2}$. [3]

For the rest of the question, use $p = 0.393$.

- (b) Two random samples of 5 people are chosen. Find the probability that the first sample has at least 4 people who are underweight and has more people who are underweight than the second sample. [3]

4. [VJC 18 MYE]

A shoe company manufactures a large number of sneakers every day. A small proportion, p , of those sneakers is defective. A check is carried out each day by taking a random sample of 15 sneakers and examining them for defects.

- (a) State, in context, two assumptions needed for the number of defective sneakers in the sample to be well modelled by a binomial distribution. [2]

For the rest of this question, assume that the binomial law holds.

- (b) The probability that at least 14 of the sneakers in the sample are non-defective is 0.85553. Show that the probability that at most 2 sneakers in the sample are defective is 0.972, correct to 3 decimal places. [3]

If exactly 3 sneakers are defective, a further sample of 5 sneakers is taken. A day's production is accepted as satisfactory in either one of the following cases:

- The number of defective sneakers in the sample of 15 is at most 2;
- The number of defective sneakers in the sample of 15 is 3, **and** there is no defective sneaker in the sample of 5.

- (c) Find the probability that the days' production is accepted as satisfactory. [2]

Subsequently, the manufacturer realises a fault in the check for defective sneakers. If a sneaker is defective, there is a 90% chance that the check correctly identifies it is defective. If the sneaker is not defective, there is a 5% chance that the check incorrectly identifies it as defective.

- (d) Find the probability that a sneaker is identified as not defective. [2]

5. [PJC 18 MYE]

Tomatoes are sold in boxes of 16. On average, 15% of them are damaged.

- (a) State, in context, two assumptions needed for the number of damaged tomatoes in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of damaged tomatoes in a box has a binomial distribution.

- (b) Find the most likely number of damaged tomatoes in a randomly chosen box. [2]

- (c) Find the probability that a box contains less than 6 damaged tomatoes. [1]

- (d) A box contains less than 6 damaged tomatoes. Find the probability that it contains at least 2 damaged tomatoes. [3]

- (e) Two boxes of tomatoes are randomly selected. Find the probability that one box has at least 2 damaged tomatoes and the other box has none. [2]

- (f) A market trader buys 80 boxes of tomatoes. Find the probability that there are more than 60 boxes with at least 2 damaged tomatoes, giving your answers to 4 decimal places. [3]