1. [SAJC 18 MYE (modified)]

An insurance agent tries to sell a health insurance policy to a randomly selected customer. On average, the probability that a customer will purchase the policy is 0.25. Every week, the insurance agent tries to sell the policy to n different customers. If the agent is not successful in selling the policy to a particular customer, he does not attempt to sell the policy to the same customer again. Each customer is only entitled to purchase the policy once.

(a) An insurance agent is expected to achieve a mean of at least 3 sales per week. Find the least number of customers an insurance agent should contact per week in order to meet this requirement.

A week is considered 'good' for the insurance agent if he manages to sell the policy to at least 4 customers during the week. It is given that the insurance agent contacts 20 randomly selected customers per week. Find

(b) the probability that a randomly selected week is considered 'good',

(c) the probability that in a year of 52 working weeks, the insurance agent has between 24 to 36 'good' weeks inclusive.

2. [IJC 18 MYE (modified)]

A large number of people attend the movie premiere of 'Revenger: Finite War' and each of them receives a free hamper. There are two types of hampers, either Type-A or Type-B. On average, 15% of the people receive Type-A hampers. A random sample of 10 people is chosen and the number of people who receive Type-A hampers is the random variable A.

- (a) Find the probability that there are less than 3 people who receive Type-A hampers.
- (b) Given that at least 7 people receive Type-B hampers, find the probability that at least 2 people receive Type-A hampers.
- (c) Find the modal number of people receiving Type-B hampers.

Some of the hampers are considered 'lucky' hampers as they each contain an additional gift of movie vouchers. It is known that 70% of Type-A hampers are 'lucky' hampers, while 50% of Type-B hampers are 'lucky' hampers.

(d) Given that Tony receives a 'lucky' hamper, find the probability that it is a Type-A hamper.

[3]

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3. [VJC 18 MYE]

A shoe company manufactures a large number of sneakers every day. A small proportion, p, of those sneakers is defective. A check is carried out each day by taking a random sample of 15 sneakers and examining them for defects.

(a) State, in context, two assumptions needed for the number of defective sneakers in the sample to be well modelled by a binomial distribution.

[2]

For the rest of this question, assume that the binomial law holds.

(b) The probability that at least 14 of the sneakers in the sample are non-defective is 0.85553. Show that the probability that at most 2 sneakers in the sample are defective is 0.972, correct to 3 decimal places.

[3]

If exactly 3 sneakers are defective, a further sample of 5 sneakers is taken. A day's production is accepted as satisfactory in either one of the following cases:

- The number of defective sneakers in the sample of 15 is at most 2;
- The number of defective sneakers in the sample of 15 is 3, and there is no defective sneaker in the sample of 5.
- (c) Find the probability that the days' production is accepted as satisfactory.

[2]

Subsequently, the manufacturer realises a fault in the check for defective sneakers. If a sneaker is defective, there is a 90% chance that the check correctly identifies it is defective. If the sneaker is not defective, there is a 5% chance that the check incorrectly identifies it as defective.

(d) Find the probability that a sneaker is identified as not defective.

[2]