

1. [RI Prelims 17 (modified)]

For events X and Y , it is given that $P(X|Y) = \frac{1}{2}$, $P(Y|X) = \frac{2}{3}$ and $P(X \cup Y) = \frac{5}{6}$.

Find

- (a) $P(X)$, [3]
- (b) $P(X \cup Y')$. [2]
- (c) Explain whether X and Y are independent. [1]

2. [AJC MYE 18]

A student has two routes to get to school. The probability that he chooses the first route on any day is p where $0 < p < 1$, and the probability of him being late for school is 0.1 if he chooses the first route and 0.2 for the second route. There is a probability of 0.14 that a student is late for school on any given day.

- (a) Find the value of p . [2]
- (b) Given that he is not late for school, find the probability that he chose the first route. [2]
- (c) Find the probability that the student is late for school more than once in three days. [2]

3. [EJC MYE 18 (modified)]

A company sells bicycles and conducted a market survey among the users to understand more about consumers' behaviour. The results showed that among all bicycles, 60% belongs to males and the rest to females. 90% of the bicycles belonging to the males and 70% of those belonging to the females are racers (a type of bicycle). A bicycle is chosen at random. Find the probability that it

- (a) is a racer or that it belongs to a female (or both), [2]
- (b) belongs to a female given that it is not a racer. [3]

Two bicycles are chosen at random. Find the probability that one is a racer and one is not. [2]

4. [DHS MYE 18]

During teatime every afternoon, Miss Chocoholic lays 5 plates on the dining table, with each plate containing 1, 2, 3, 4 and 5 pieces of chocolates respectively.

In hope of reducing her chocolate addiction, Miss Chocoholic undergoes the following ritual:

- I. Cover all the plates and select 3 plates at random.
- II. From the selected plates, discard the plates with the greatest and least number of chocolates.
- III. Consume all the chocolate in the remaining plate.

Let X denote the number of chocolates Miss Chocoholic consumes in an afternoon.

- (a) Show that $P(X = 3) = 0.4$ and find the probability distribution of X . [3]
- (b) Find $E(X)$ and show that $Var(X) = 0.6$. [2]
- (c) Find the probability that Miss Chocoholic consumes a mean number of 3 chocolates in 2 randomly chosen afternoons. [3]

5. [JJC MYE 18]

A discrete random variable X takes values 1, 3, 5, 6 with probabilities as shown in the table.

x	1	3	5	6
$P(X = x)$	$\frac{k}{3}$	k	$\frac{5k}{2}$	$\frac{3k}{4}$

- (a) Find k , leaving your answer as a fraction. [2]
- (b) Find $E(|\sin X|)$. [2]

X_1 and X_2 are two independent observations of X .

- (c) Show that $P(X_1 + X_2 = 7) = \frac{72}{3025}$. [1]
- (d) Find $P(X_1 - X_2 = 2)$. [3]