# Integration – Exercise 1

## (a) Techniques of Integration – Indefinite Integral

1. Integrate the following with respect to $x$.

(a) $2x^3$ (b) $2 + x$

(c) $3x^2 - 2x + 3$ (d) $\dfrac{1}{2x^5}$

(e) $\dfrac{3}{x^2}$ (f) $x^2 + \dfrac{1}{\sqrt{x}}$

(g) $x\left(1+\dfrac{1}{x^3}\right)$ (h) $\dfrac{x^2+1}{2x^2}$

(i) $\sqrt{x}(\sqrt{x}+1)$ (j) $(2x-\sqrt{x})^2$

2. Integrate the following with respect to $x$.

(a) $(2 + 3x)^3$ (b) $(2x + 5)^4$

(c) $(1 - 3x)^5$ (d) $(9x + 1)^{-3}$

(e) $\dfrac{1}{(2x-7)^3}$ (f) $\dfrac{2}{\sqrt{3-4x}}$

(g) $\dfrac{4}{3\sqrt{6x-1}}$

**Notes:**

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c,\ n \neq -1$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1,\ a \neq 0$$

**Examples**

(a) $\int 3x^5 dx = \frac{3x^6}{6} + c$

(b) $\int (x^3 + 3x + 2)\,dx = \frac{x^4}{4} + \frac{3x^2}{2} + 2x + c$

(c) $\int (2x+3)^{10} dx = \frac{(2x+3)^{11}}{11(2)} + c$

## (b) Techniques of Integration – Definite Integral

3. Evaluate the following definite integrals.

(a) $\displaystyle\int_{-2}^{1} (8x-4)dx$

(b) $\displaystyle\int_{1}^{8} \frac{1}{2}x^{-\frac{1}{2}}\,dx$

(c) $\displaystyle\int_{1}^{4} (6x-3\sqrt{x})dx$

(d) $\displaystyle\int_{-1}^{0} (3x^2-2x+5)dx$

(e) $\displaystyle\int_{1}^{4} \left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)dx$

(f) $\displaystyle\int_{1}^{2} \left(x^2-\frac{4}{x^2}\right)dx$

[(a) – 24 (b) 1.83 (c) 31 (d) 7 (e) $\frac{2}{3}$ (f) $\frac{1}{3}$]

**Example**

$\int_1^4 (3x+5)\,dx$

$= \left[\frac{3x^2}{2} + 5x\right]_1^4$

$= \left[\frac{3(4)^2}{2} + 5(4)\right] - \left[\frac{3(1)^2}{2} + 5(1)\right]$

**Notes:**

(a) $\int_a^a f(x)dx = 0$

(b) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(c) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

**(c) Integration as the Reverse Process of Differentiation**

4. Find the equation of the curve which passes through the point (2, 4) and for which $\frac{dy}{dx} = x(3x - 1)$.

[$y = x^3 - \frac{1}{2}x^2 - 2$]

5. The gradient of a curve is $6(4x - 1)^2$ and the curve passes through the origin. Find the equation of the curve.

[$y = \frac{1}{2}(4x - 1)^3 + \frac{1}{2}$]

6. The curve for which $\frac{dy}{dx} = 3x^2 + k$, where k is a constant, has a turning point of (–2, 6). Find
   (a) the value of *k*
   (b) the equation of the curve.

[(a) –12 (b) $y = x^3 - 12x - 10$]

7. The curve for which $\frac{dy}{dx} = 2x + a$ where *a* is a constant is such that the normal at (1, 5) cuts the *x*-axis at (6, 0). Find the value of *a* and the equation of the curve.

[$a$ = –1, $y = x^2 - x + 5$]

8. The curve for which $\frac{dy}{dx} = kx - 5$, where *k* is a constant is such that the tangent at (2, 2) passes through the origin. Determine
   (a) the value of *k*
   (b) the equation of the curve.

[(a) 3 (b) $y = \frac{3}{2}x^2 - 5x + 6$]

9. Given that $\frac{d}{dx}\ln(x+1) = \frac{1}{x+1}$. Hence evaluate $\int_1^2 \frac{1}{x+1}dx$.

[$\ln\frac{3}{2}$]

10. Show that $\frac{d}{dx}(x\sqrt{1+2x^2}) = \frac{4x^2+1}{\sqrt{2x^2+1}}$. Hence evaluate $\int_0^2 \frac{4x^2+1}{\sqrt{2x^2+1}}dx$.

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11. Show that $\frac{d}{dx}\left(x\sqrt{x+1}\right) = \frac{3x+2}{2\sqrt{x+1}}$. Hence evaluate $\int_3^8 \frac{3x+2}{\sqrt{x+1}}dx$.

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12. Show that $\frac{d}{dx}\left(\frac{x}{1+2x}\right) = \frac{1}{(1+2x)^2}$. Hence evaluate $\int_1^4 \left(\frac{2}{1+2x}\right)^2 dx$.

[$\frac{4}{9}$]

13. Show that $\frac{d}{dx}\tan^3 x = 3\tan^2 x \sec^2 x$ and hence evaluate $\int_0^{\frac{\pi}{4}} \left(\frac{\tan x}{\cos x}\right)^2 dx$.

[$\frac{1}{3}$]

14. Given that $y = x\ln x$, find an expression for $\frac{dy}{dx}$.
    Hence find $\int \ln x \, dx$.

[$y = x\ln x - x + c$ ]

15. Given that $y = x^2\ln x$, find an expression for $\frac{dy}{dx}$.
    Hence find $\int x\ln x \, dx$.

[$y = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$]

16. Given that $y = x^3\ln x$, find an expression for $\frac{dy}{dx}$.
    Hence find $\int x^2\ln x \, dx$.

[$y = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + c$]

17. Given that $\int_0^1 f(x)dx = \int_1^3 f(x)dx = 3$, evaluate
    (a) $\int_0^1 f(x)dx + \int_3^1 f(x)dx$ (b) $\int_1^0 f(x)dx$
    (c) $\int_1^3 [2f(x) + 5]dx$

[(a) 0 (b) –3 (c) 16]