Integration – Exercise 2

(a) Techniques of Integration – Trigonometric Functions

1. Integrate the following with respect to x.

(a) $\sin x + 2$
(b) $2 - \cos x$
(c) $\cos x - \sin x + 3x^2$
(d) $5\sec^2 x - 4\sin x$
(e) $\cos 2x$
(f) $\sin 3x$
(g) $\cos\frac{1}{6}x + \sin 4x$
(h) $3\cos(2x + 1)$
(i) $\sin\left(2x + \frac{\pi}{2}\right)$
(j) $\cos\left(3x - \frac{\pi}{2}\right) - 4\sin(2 - x)$
(k) $-3\sin(2 - x)$
(l) $\sec^2(2x + \pi)$
(m) $\tan^2(2x)$
(n) $\sin^2 x$
(o) $\cos^2 x$
(p) $2\sin x\cos x$
(q) $\sin 2x\cos 2x$
(r) $\frac{1}{\cos^2 2x}$

2. Evaluate the following definite integrals.

(a) $\int_0^{\frac{\pi}{6}} \cos x \, dx$
(b) $\int_0^{\frac{\pi}{4}} \sin 2x \, dx$
(c) $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$
(d) $\int_0^{\frac{\pi}{2}} (1 - 2\sin x) \, dx$
(e) $\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \cos\left(2x + \frac{\pi}{3}\right) dx$
(f) $\int_{\frac{\pi}{2}}^{\pi} 2\sin(\pi - x) \, dx$
(g) $\int_0^{\frac{\pi}{4}} (1 + \tan^2 x) \, dx$
(h) $\int_0^{\frac{\pi}{4}} \frac{1 + \cos^2 x}{\cos^2 x} \, dx$

Note:

(a) $\int \cos(ax + b)\,dx = \frac{1}{a}\sin(ax + b) + c$

(b) $\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b) + c$

(c) $\int \sec^2(ax + b)\,dx = \frac{1}{a}\tan(ax + b) + c$

Useful formulas

(a) $\cos^2 x = \frac{1 + \cos 2x}{2}$
(b) $\sin^2 x = \frac{1 - \cos 2x}{2}$
(c) $\sin 2x = 2\sin x\cos x$
(d) $\sec^2 x = \tan^2 x + 1$
(e) $\frac{1}{\cos^2 x} = \sec^2 x$

Methods of integrating Trigonometric functions

(i) Use identity

$$\int \tan^2(ax + b)\,dx = \int [\sec^2(ax + b) - 1]dx = \frac{\tan(ax+b)}{a} - x + c$$

$$\int \frac{1}{\cos^2(ax+b)}dx = \int \sec^2(ax + b)\,dx = \frac{\tan(ax+b)}{a} + c$$

(ii) Use double angle formula

$$\int \sin^2(ax + b)\,dx = \int \left[\frac{1 - \cos 2(ax+b)}{2}\right] dx = \frac{1}{2}\left[x - \frac{\sin 2(ax+b)}{2a}\right] + c$$

Examples

(a) $\int \sin(2x + \pi)\,dx = -\frac{1}{2}\cos(2x + \pi) + c$

(b) $\int \tan^2(4x)\,dx = \int(\sec^2 4x - 1)dx = \frac{1}{4}\tan(4x) - x + c$

(b) Techniques of Integration – Exponential Functions

3. Evaluate the following definite integrals.

(a) $\int_0^2 e^x \, dx$ (b) $\int_0^1 e^{2x} \, dx$

(c) $\int_0^2 e^{-\frac{1}{2}x} \, dx$ (d) $\int_1^2 e^{1-x} \, dx$

(e) $\int_0^{\ln 2} e^{3x} \, dx$ (f) $\int_0^{\ln 3} e^x \, dx$

[(a) $e^2 - 1$ (b) $\frac{1}{2}(e^2 - 1)$ (c) $2\left(1 - \frac{1}{e}\right)$ (d) $\left(1 - \frac{1}{e}\right)$ (e) $\frac{7}{3}$ (f) 2]

4. Show that $\int_1^2 e^{3x-2} dx = 17.3$

5. Find

(a) $\int (2 + e^x)^2 dx$ (b) $\int \frac{e^{3x}-2}{e^x} dx$

[(a) $4x + 4e^x + \frac{1}{2}e^{2x} + c$ (b) $\frac{1}{2}e^{2x} + 2e^{-x} + c$]

6. Find $\frac{d}{dx}\left(e^{\sqrt{x}}\right)$ and hence evaluate $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

[2(e –1)]

Note: $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$

(c) Techniques of Integration – Logarithmic Functions

7. Integrate the following with respect to *x*.

(a) $\int \frac{3}{x} \, dx$ (b) $\int \frac{1}{x+1} \, dx$

(c) $\int \frac{1}{2x-1} \, dx$ (d) $\int \frac{3}{2x+1} \, dx$

(e) $\int \frac{2}{1-2x} \, dx$ (f) $\int \frac{1}{4-3x} \, dx$

8. Integrate the following with respect to *x*.

(a) $\int \frac{2x-1}{(x+1)(x+2)} \, dx$ (b) $\int \frac{1}{(1-x)(3-2x)} \, dx$

[(a) $-3\ln(x+1) + 5\ln(x+2) + c$ (b) $\ln(3-2x) - \ln(1-x) + c$]

9. Evaluate the following definite integrals.

(a) $\int_3^5 \frac{1}{(x-2)(x-1)^2} \, dx$ (b) $\int_3^5 \frac{1}{(x+3)(x-2)^2} \, dx$

[(a) $\ln\left(\frac{3}{2}\right) - \frac{1}{4}$ (b) $\frac{1}{25}\left[2\ln\left(\frac{2}{3}\right) + \frac{10}{3}\right]$]

Note:

(a) $\int \frac{1}{ax+b} dx = \frac{1}{a}\ln(ax+b) + c$

(b) **Integration of $\frac{1}{(ax+b)(cx+d)}$ -----use partial fractions!**

Example

$\int \frac{4x+1}{(x+1)(x-2)} dx \quad = \int \left[\frac{1}{x+1} + \frac{3}{x-2}\right] dx$

$= \ln(x+1) + 3\ln(x-2) + c$