

Normal Distribution

1. (a) The plums sold by a supermarket are graded 'small', 'medium' or 'large'. The masses of the plums have a normal distribution. Plums with a mass less than 22 grams are graded as small, plums with a mass greater than 29 grams are graded as large and the rest are graded as medium. Given that 30% of plums are small and 20% are large, find the mean and standard deviation of the distribution. [4]
- (b) The masses, in kilograms, of apples and nectarines sold by the supermarket have independent normal distributions with means and standard deviations as shown in the following table.

| | Mean | Standard deviation |
|------------|------|--------------------|
| Apples | 0.15 | 0.03 |
| Nectarines | 0.07 | 0.02 |

- (i) Two apples and four nectarines are chosen at random. Find the probability that the total mass of the two apples is greater than the total mass of the four nectarines. [4]
- (ii) Apples cost \$9 per kilogram and nectarines cost \$12 per kilogram. Find the mean and the variance of the total cost of two apples and four nectarines and hence find the probability that the total cost is between \$5 and \$6. [5]

(2009/P1/Q12)

2. Sweets of a certain brand are individually wrapped. The masses, in grams, of the unwrapped sweets and the wrappers have independent normal distributions with means and standard deviations as shown in the table below.

| | Mean | Standard deviation |
|------------------|------|--------------------|
| Unwrapped sweets | 40 | 3 |
| Wrappers | 4 | 0.5 |

- (i) Find the probability that an individual unwrapped sweet has mass less than 36 grams. [1]
- (ii) State the mean and variance of the mass of an individual **wrapped** sweet. Find the probability that a wrapped sweet has mass between 42 grams and 46 grams. [3]

Twelve wrapped sweets are packed together in a cardboard tube. The mass of an empty tube is normally distributed with mean 50 grams and standard deviation 5 grams. The masses of all sweets and tubes are independent.

- (iii) Find the probability that the total mass of a tube containing 12 wrapped sweets is more than 600 grams, stating clearly the mean and variance of the distribution that you use. [4]

A rival company produces similar tubes of sweets. The masses of these tubes of sweets have a normal distribution. Over a long period of time, it is found that 5% of them have a mass less than 450 grams and 8% have a mass more than 550 grams.

- (iv) Find the mean and variance of this distribution. [5]

(2010/P1/Q12)

3. Boys and girls visiting a theme park have masses, in kg, that are independent and are normally distributed with means and standard deviations as shown in the following table.

| | Mean mass | Standard deviation |
|-------|-----------|--------------------|
| Boys | 60 | 12 |
| Girls | 50 | 10 |

(i) Find the probability that the mass of a boy chosen at random is between 50 kg and 70 kg. [2]

(ii) A boy and a girl are chosen at random. Find the probability that the mass of the boy is greater than the mass of the girl, stating clearly the mean and variance of the distribution that you use. [4]

On a ride at the theme park, trains carrying up to 5 people travel around a track. The total mass of the people on the train must not exceed the safety limit of 300 kg.

(iii) Three boys and two girls are chosen at random. Find the probability that their total mass is less than 300 kg, stating clearly the mean and variance of the distribution that you use. [4]

The track is improved and new trains carrying up to 6 people are designed. The new safety limit is L kg.

(iv) Obtain the equation for L , given that it is 95% certain that 6 boys chosen at random have a total mass not exceeding L kg. Hence find L . [3]

(2011/P1/Q12)

4. A supermarket sells two types of grapefruit, A and B . The masses, in kilograms, of the grapefruit of each type have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

| | Mean (kg) | Standard deviation (kg) | Selling price (\$ per kg) |
|--------|-----------|-------------------------|---------------------------|
| Type A | 0.25 | 0.02 | 1.50 |
| Type B | 0.35 | 0.03 | 2.40 |

Stating clearly the mean and variance of all distributions that you use, find the probability that

(i) the total mass of 10 randomly chosen grapefruit of type A is less than 2.4 kg, [3]

(ii) the total mass of 6 randomly chosen grapefruit of type A is within 0.2 kg of the total mass of 5 randomly chosen grapefruit of type B . [4]

Mrs Woo buys 3 grapefruit of type A and 3 grapefruit of type B . Mr Tan buys 10 grapefruit of type A .

(iii) Stating clearly the mean and variance of the distribution that you use, find the probability that Mrs Woo pays more than Mr Tan. [6]

(2012/P1/Q12)

5. A pet shop sells two types of animal food. Type *A* is supplied by a manufacturer and sold in packets with the food content having a mean mass of 1 kg. The masses of the food content are normally distributed. It is known that 20% of the packets contain less than 990 g of food.

(i) Find the standard deviation of the distribution. [3]

Type *B* animal food is mixed by the shop owner from two ingredients *P* and *Q*. One packet contains 3 scoops of ingredient *P* and 2 scoops of ingredient *Q*. The masses, in grams, of the food in scoops of ingredients *P* and *Q* have independent normal distributions with means and standard deviations as shown in the following table.

| | Mean | Standard deviation |
|---------------------|------|--------------------|
| Ingredient <i>P</i> | 240 | 10 |
| Ingredient <i>Q</i> | 145 | 8 |

(ii) Find the probability that a randomly selected packet of Type *B* has a mass of food less than 1 kg. State the mean and variance of any distribution that you use. [4]

(iii) Find the probability that the mass of food in a randomly selected packet of Type *B* is more than the mass of food in a randomly selected packet of Type *A*. State the mean and variance of any distribution that you use. [4]

(2013/P1/Q11)

6. The heights of girls in a school have a normal distribution with mean 142.2 cm and standard deviation 6 cm. Find the probability that a girl chosen at random from this school has height

(i) less than 146 cm, [2]

(ii) within 5 cm of the mean. [2]

(2014/P1/Q6)

7. The outputs of a certain metal, in tonnes, extracted each day from two mines, *A* and *B*, have independent normal distributions. The mean of the distribution of the daily output from *A* is 50 tonnes. The probability that the daily output from *A* is more than 75 tonnes is 0.0189.

(i) Show that the variance of this distribution is 145 tonnes², correct to 3 significant figures. [3]

The mean and variance of the distribution of the daily output from *B* are 75 tonnes and 64 tonnes² respectively. *B* operates for seven days each week.

(ii) Find the probability that in a 7-day week the output from *B* is less than 500 tonnes. [3]

A operates for five days each week.

(iii) Find the probability that in any particular week the output from *B* is more than twice the output from *A*. You should state the mean and variance of any distribution that you use. [5]

(2014/P1/Q12)

8. The masses of peaches sold by a shop have a normal distribution. Over a long period of time, it is found that 20% of peaches have a mass less than 40 grams and 25% of peaches have a mass greater than 60 grams. Find the mean and variance of the distribution. [4]

(2015/P1/Q6)

9. Men and women staying at a large hotel have masses, in kg, that are normally distributed with means and standard deviations as shown in the following table.

| | Mean mass | Standard deviation |
|-------|-----------|--------------------|
| Men | 77 | 9.8 |
| Women | 62 | 10.6 |

- (i) Find the probability that the mass of a man chosen at random is within ± 2 kg of the mean mass of men. [2]
- (ii) Find the probability that the total mass of three men chosen at random is greater than the total mass of four women chosen at random. State the mean and variance of the distribution that you use. [4]

The lift in the hotel has a safety limit of 460 kg. Three men and four women are chosen at random.

- (iii) Find the probability that they can safely travel in the lift together. State the mean and variance of the distribution that you use. [3]

(2015/P1/Q11)

10. Shortbread biscuits of a certain brand are sold in boxes containing 12 biscuits. The masses, in grams, of the individual biscuits and of the empty boxes have independent normal distributions with means and standard deviations as shown in the following table.

| | Mean | Standard deviation |
|--------------------|------|--------------------|
| Individual biscuit | 20 | 1.1 |
| Empty box | 5 | 0.8 |

- (i) Find the probability that the mass of an individual biscuit is less than 19 grams. [2]
- (ii) Find the probability that the total mass of a box containing 12 biscuits is more than 248 grams. State the mean and variance of the distribution that you use. [4]

The cost of producing biscuits is 0.6 cents per gram and the cost of producing empty boxes is 0.2 cents per gram.

- (iii) Find the probability that the total cost of producing a box containing 12 biscuits is between 142 cents and 149 cents. State the mean and variance of the distribution that you use. [5]

(2016/P1/Q12)

11. As part of an assessment of the health of people in a particular country, the heights of a large number of adult males have been recorded. The results show that 20% of them have a height less than 1.6 m and 30% of them have a height greater than 1.75 m. Assuming that the heights of adult males are normally distributed, find the mean and variance of the distribution. [4]

(2017/P1/Q6)

12. There are bus and train services between the towns of Ayton and Beeton. The journey times, in minutes, by bus and by train have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

| | Mean | Standard deviation |
|-------|------|--------------------|
| Bus | 45 | 4 |
| Train | 42 | 3 |

- (i) Find the probability that a randomly chosen bus journey takes less than 48 minutes. [1]
- (ii) Find the probability that two randomly chosen bus journeys each take more than 48 minutes. [2]
- (iii) The probability that the total time for two randomly chosen bus journeys is more than 96 minutes is denoted by p . Without calculating its value, explain why p will be greater than your answer to part (ii). [1]

Lan lives in Ayton and works in Beeton. Three days a week he travels from home to work by bus and two days a week he travels from home to work by train.

- (iv) Find the probability that for 3 randomly chosen bus journeys and 2 randomly chosen train journeys, Lan's total journey time is more than 210 minutes. [4]

Journeys are charged by the time taken. For bus journeys the charge is \$0.12 per minute and for train journeys the charge is \$0.15 per minute.

Let B represent the cost of one journey from Ayton to Beeton by bus.

Let T represent the cost of one journey from Ayton to Beeton by train.

- (v) Find $P(3B - 2T < 3)$ and explain, in the context of this question, what your answer represents. [5]

(2017/P1/Q12)