S1. Probability Summary Notes

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Last updated February 24, 2019.
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1.1 Notation and basics.

- For this set of notes, $A, B \dots$ will be used to denote **events**.
- $P(A) = \dfrac{\text{No. of ways for } A \text{ to occur}}{\text{Total no. of possible outcomes}}$.
- For all events A, $0 \le P(A) \le 1$.
- **[The addition principle]**
  If event A is made up entirely of **either** X **or** Y (with no overlap), with probabilities x, y respectively, then
  $$P(A) = x + y.$$
  X and Y are sometimes called **cases**.
- **[The multiplication principle]**
  If event A can be broken up into two steps: **first** step X with probability x, **then** step Y with conditional probability y, then
  $$P(A) = xy.$$
- **[The complementation principle]**
  If A' is used to denote all outcomes that are **not** in A, then
  $$P(A) = 1 - P(A').$$

1.2 Two important formulas.

- The **conditional probability** $P(A|B)$ is the probability that A occurs **given that** B has occurred.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)} \tag{1}$$
$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}} \tag{2}$$

1.3 Some "Math-to-English translations".

Notation	*Probability of*
$P(A \cap B)$	A **and** B
$P(A')$	**not** A
$P(A \cup B)$	A **or** B (or both)
$P(A\|B)$	A **given** B A **if** B B has occurred. What is the probability of A?

1.4 Some common problem-solving tools.

- The **Venn diagram** is useful for visualizing sets, especially when the set notation gets a bit complicated. (e.g. $P(A' \cap B')$.) It also allows us to derive various useful set identities.
- The **tree diagram** is useful for problem sums involving events where one prior event affects the probability of a second. When labeling a tree diagram, note that each set of "branches" should add up to one. Also, conditional probabilities are used in the middle of the tree.
- A **table of outcomes** is useful for problems involving some combination of two events. For example, if we are interested in the sum of two separate die rolls, we can list the possible outcomes of each die on the rows/columns.

1.5 Mutually exclusive and/or independent events.

- Events A and B are **mutually exclusive** if they cannot occur at the same time. Mathematically, $P(A \cap B) = 0$ for mutually exclusive events.
- Events A and B are **independent** if the occurrence of A does not affect the probability of B (and vice versa). Mathematically,

$$P(A|B) = P(A) \tag{3}$$

$$P(A \cap B) = P(A)P(B) \tag{4}$$

- REMARK: Formula (4) should only be used for independent events. It is not true all the time.
- REMARK: Mutually exclusivity and independence are separate concepts.