1. **[CJC Prelims 18]**
For events A and B, it is given that $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{5}$ and $P(A' \cap B') = \frac{1}{6}$.
Find $P(A|B')$. [3]

2. **[SRJC Prelims 18]**
For events A and B, it is given that $P(A) = \frac{11}{20}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{4}$.
   (a) Find the probability where event A occurs or even B occurs but not both. [1]
   (b) Find the probability where event B has occurred but not A. [1]

   For a third event C it is given that $P(C) = \frac{1}{2}$ and A and C are independent.
   (c) Find the range of values of $P(A' \cap B' \cap C)$. [3]

3. **[SAJC Prelims 18]**
For two mutually exclusive events A and B, it is given that $P(A) = 0.65$ and $P(B|A') = \frac{2}{7}$.
   (a) Show that $P(B) = 0.1$. [2]

   For a third event C, it is given that $P(A \cap C) = 0.39$.
   (b) Find $P(C'|A)$. [2]

   It is given that B and C are independent and $P(A' \cap B' \cap C) = 0.15$.
   (c) Find $P(B \cap C)$. [2]
   (d) Hence or otherwise, determine whether the events A and C are independent. [1]

4. **[AJC Prelims 17]**
A vehicle insurance company classifies the drivers it insures as class A, B and C according to whether they are of low risk, medium risk or high risk with regard to having an accident. The company estimates that 30% of the drivers who are insured are class A and 50% are class B. The probability that a class A driver will have at least one accident in any 12 month period is 0.01, the corresponding probabilities for class B and C are 0.03 and 0.06 respectively.
   (a) Find the probability that a randomly chosen driver will have at least one accident in a 12-month period. [2]
   (b) The company sold a policy to a driver and within 12 months, the driver had at least one accident. Find the probability that the driver is of class C. [2]
   (c) Three drivers insured by the company are chosen randomly. Find the probability that all three drivers are of class C and exactly one of them had at least one accident in a 12-month period. [3]

5. **[DHS Prelims 17]**
The insurance company Adiva classifies 10% of their car policy holders as low risk, 60% as average risk and 30% as high risk. Its statistical database has shown that of those classified as low risk, average risk and high risk, 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that

(a) a randomly chosen policy holder is not involved in any accident if the holder is classified as 'average risk', [1]

(b) a randomly chosen policy holder is not involved in any accident, [2]

(c) a randomly chosen policy holder is classified as 'low risk' if the holder is involved in at least one accident. [2]

** It is known that the cost of repairing a car when it meets with an accident has the following probability distribution.

It is known that a low risk policy holder will not be involved in more than one accident in a year. You may assume that there will be no cost incurred by the company in insuring a holder whose car is not involved in any accident.

(d) ** Construct the probability distribution table of the cost incurred by Adiva in insuring a low risk policy holder assuming that the cost of repairing a car is independent of a low risk policy holder meeting an accident. [1]

(e) ** In order to have an expected profit of $200 from each policy holder, find the amount that Adiva should charge a low risk policy holder when he renews his annual policy. [2]

6. **[HCI Prelims 17]**
A man wishes to buy a 4-digit number lottery. He plays by randomly choosing any number from 0000 to 9999. It is assumed that each number is equally likely to be chosen. Find the probability that a randomly chosen 4-digit number has

(a) four different digits, [1]

(b) exactly one of the first three digits is the same as the last digit, and the last digit is even, [3]

(c) four different digits with the first digit greater than 6, given that the 4-digit number has odd and even digits that alternate. [4]

7. **[IJC Prelims 17]**
Seven red counters and two blue counters are placed in a bag. All the counters are indistinguishable except for their colours. Clark and Kara take turns to draw a counter from the bag at random with replacement. The first player to draw a blue counter wins the game and the game ends immediately.

If Clark draws first, find the probability that

(a) Clark wins the game at his third draw, [2]

(b) Kara wins the game. [3]

8. **[TPJC Prelims 17]**
The number of employees of a statutory board, classified by department and years of working experience, is shown below.

	5 years or less	5 to 10 years	10 years or more	Total
Human Resource Department	20	50	30	100
Legal Department	15	60	45	120
Finance Department	25	30	45	100
Total	60	140	120	320

The Managing Director of the statutory board wishes to select three employees to participate in an overseas conference. The Managing Director selects one employee from each department to participate in the conference.

(a) Find the probability that two of the selected employees have years of working experience 10 years or more and the remaining one has years of working experience 5 years or less. [3]

(b) Given that exactly one of the selected employees has years of working experience 5 years or less, find the probability that one of the selected employees is from the Legal Department and has years of working experience 5 to 10 years. [3]

9. **[VJC Prelims 17]**
John and Peter play a game of chess. It is equally likely for either player to make the first move. If John makes the first move, the probability of him winning the game is 0.3 while the probability of Peter winning the game is 0.2. If Peter makes the first move, the probability of him winning the game is 0.5 while the probability of John winning the game is 0.4. If there is no winner, then the game ends in a draw.

(a) Find the probability that Peter made the first move given that he won the game. [3]

(b) John and Peter played a total of three games. Assuming that the results of the three games are independent, find the probability that John wins exactly one game. [3]

10. **[NYJC Prelims 18]**
During a Mathematics lesson, Ms Kim wanted her pupils to build a model. She carried two indistinguishable boxes of bricks into class. The bricks are identical and indistinguishable except for colours. The number of coloured bricks found in each box is as follows.

Colour of Bricks	Box 1	Box 2
Blue	2	4
Red	3	3
Yellow	5	3

A pupil, Donald, has to draw two bricks from these boxes randomly to build a model. He draws a brick randomly from one of the boxes. The brick is not replaced. He then draws a second brick randomly from one of the boxes.

[Question continued on the next page]

(a) Show that the probability that Donald does not draw any yellow brick brick is $\frac{25}{72}$. [2]

(b) Find the probability that Donald draws two yellow bricks. [3]

(c) ** Tabulate the probability distribution table for the number of yellow bricks drawn and find the expected number of yellow bricks drawn by Donald. [2]

Ms Kim decided to combine the two boxes of bricks into a single box.

(d) ** Donald draws 5 bricks **with replacement** from the single box. Find the probability that he draws less than two yellow bricks. [2]

(e) Donald wanted to build the model with a yellow brick. He randomly draws a brick from the single box one at a time **with replacement** until he gets a yellow brick. Find the probability that it will not take him more than nine draws to get a yellow brick. [3]

11. [**MI Prelims 17**]
In a survey conducted by the National Eye Centre, it was found that $p\%$ are boys and the remaining are girls. The probability that a randomly chosen boy wears spectacles is 0.3 and the probability that a randomly chosen girl wears spectacles is 0.24.

(a) Find the value of p, given that the probability that a randomly chosen child wears spectacles is 0.267. [2]

(b) For a general value of p, the probability that a randomly chosen child that wears spectacles is a girl denoted by $f(p)$. Show that $f(p) = \dfrac{4(100-p)}{400+p}$. Prove by differentiation that f is a decreasing function for $0 \le p \le 100$, and explain what this statement means in the context of the question. [5]

12. [**ACJC Prelims H1 17**]
In a game of penalty kicks, a player is given three attempts at scoring. Once the player scores, he wins and the game ends. Henry, who has a 0.7 chance of scoring on any penalty kick, plays the game.

(a) Draw a probability tree diagram to illustrate one such game. [1]

Find the probability that in one game, Henry

(b) scores on the second attempt, [1]

(c) made two attempts, given that he wins. [2]

In 3 such games, find the probability that Henry scores on the first attempt in exactly on game, and on the second attempt in exactly one game. [2]

13. **[JJC Prelims H1 17]**
A fair six-sided die is tossed once. If the score on the die is 1 or 2, a ball is picked from bag A. If the score on the die is 3, 4, 5 or 6, a ball is picked from bag B. Bag A contains 6 red and 4 blue balls. Bag B contains 5 red, 3 blue and 2 green balls. Events A and R are defined as follows:

$$\text{Event } A = \{\text{A ball is picked from bag } A\}$$
$$\text{Event } B = \{\text{A red ball is picked}\}$$

Find

(a) $P(R)$, [2]

(b) $P(A'|R)$. [3]

(c) State with a reason whether events A and R are independent. [1]

14. **[MJC Prelims H1 17]**

(a) A and B are events such that $P(A|B') = \frac{4}{17}$, $P(B) = \frac{23}{40}$ and $P(A \cap B) = \frac{3}{8}$.

i. By using a Venn diagram or otherwise, find $P(A' \cap B')$. [3]

ii. Determine if A and B are independent events. [2]

(b) A basket contains 35 durians, of which 15 are MSW durians and 20 are D24 durians. Of the MSW durians, 4 are infested with maggots and of the D24 durians, 3 are infested with maggots. Two durians are chosen at random from the basket.

i. Show that the probability that both are MSW durians and at least one durian is infested with maggots is $\frac{10}{119}$. [2]

ii. Given that at least one durian is infested with maggots, find the probability that both are MSW durians. [3]

15. **[YJC Prelims H1 17]**
The probability that a train service breaks down is $\frac{1}{15}$. When the train service functions normally, 5% of the people travelling to work by train are late. It is also found that 90% of the people travelling to work by train are punctual.

(a) Draw a tree diagram to represent this situation, showing all possible outcomes and the associated probabilities. [3]

(b) Given that a randomly chosen person travelling to work by train is late, find the probability that the train service functions normally. [2]

Answers

1. $\frac{3}{4}$.
2. (a) $\frac{9}{20}$.
   (b) $\frac{3}{20}$.
   (c) $\frac{3}{40} \leq P(A' \cap B' \cap C) \leq \frac{9}{40}$.
3. (b) 0.4.
   (c) 0.06.
   (d) Independent.
4. (a) 0.03.
   (b) 0.4.
   (c) 0.00127.
5. (a) 0.85.
   (b) 0.834.
   (c) 0.00602.
   (d) $P(C = 0) = 0.99, P(C = 5) = 0.0075, P(C = 10) = 0.0015, P(C = 50) = 0.0008, P(C = 100) = 0.0002$.
   (e) \$312.50.
6. (a) $\frac{63}{125}$.
   (b) $\frac{243}{2000}$.
   (c) $\frac{24}{125}$.
7. (a) 0.0813.
   (b) 0.4375.
8. (a) $\frac{63}{800}$.
   (b) $\frac{28}{61}$.
9. (a) $\frac{5}{7}$.
   (b) 0.444.
10. (b) $\frac{53}{360}$.
    (c) $P(X = 1) = \frac{91}{180}$, $E(X) = \frac{4}{5}$.
    (d) 0.337.
    (e) 0.990.
11. 45.

12. (b) 0.21.
    (c) $\frac{30}{139}$.
    (d) 0.0794.

13. (a) $\frac{8}{15}$.
    (b) $\frac{5}{8}$.
    (c) No since $P(A \cap R) \neq P(A) \cdot P(R)$.

14. (a) i. $\frac{13}{40}$.
        ii. Not independent.
    (b) $\frac{50}{217}$.

15. $\frac{7}{15}$.