1. **[RVHS Prelims 18]**
Two fair six-sided dice are thrown. The random variable X is the smaller of the two scores if they are different, and their common value if they are the same.

   (a) Show that $P(X = 2) = \frac{1}{4}$ and find the probability distribution of X. [2]

   (b) Hence find $E(X)$ and $Var(X)$. [2]

   A game is played with Ivan and Jon taking turns to throw the two dice each. Ivan throws the dice first and the player who first obtain the value of X equals to 2 wins the game. If Ivan wins the game, Jon pays him \$7. Otherwise, Ivan pays John \$10.

   (c) Find the player who has the higher expected gain. Justify your answer. [3]

2. **[AJC Prelims 18]**
To promote the sale of its products on Valentines Day, a shop owner gives free vouchers to its customers. To get the vouchers, customers must first participate in a game. Each customer is allowed to play the game only once.

   There are four rounds in the game. In each round, four cards are used. The message printed on one of the cards is 'Congratulations' and the message printed on the other three cards is 'Thank you'. These cards are placed facedown, and the customer would choose a card to flip in that round. Two points are scored if the card with the message 'Congratulations' is flipped. Otherwise, one point is deducted. Each customer is equally likely to choose any of the cards to flip.

   At the end of the game, the customers final score is X, where X is the sum of the points scored in the four rounds.

   The shop will then reward the customer with a voucher worth $\$(8 + 2X)$.

   (a) Show that $P(X = 2) = \frac{27}{128}$. [1]

   (b) Tabulate the probability distribution of X. [3]

   (c) Find the mean and variance of the value of the voucher a customer receives from the shop. [4]

   (d) If 100 customers visit the shop and play the game, what is the total value of the vouchers the shopkeeper is expected to give away? Estimate the probability that the total value of the vouchers given away would be more than \$650. [3]

3. **[TPJC Prelims 17]**
An unbiased six-sided die is rolled twice. The random variable X represents the higher of the two values if they are different, and their common value if they are the same. The probability distribution of X is given by the formula

$$P(X = r) = k(2r - 1) \quad \text{for } r = 1, 2, 3, 4, 5, 6.$$

(a) Find the exact value of k, giving your answer as a fraction in its simplest form. [2]

(b) Find the expectation of X. [2]

A round of the game consists of rolling the unbiased six-sided die twice, and X is taken as the score for the round. A player plays three rounds of the game.

(c) Find the probability that the total score for the three rounds is 16. [2]

4. **[TJC Prelims 17]**
In a game at the carnival, a player rolls discs onto a board containing squares, each of which bears one of the numbers 1, 2, 5 or 10. If a disc does not land within a square, the player receives nothing. The probability that the disc does not land within the square is $\frac{3}{4}$. If a disc lands within a square, the player receives the same amount (in dollars) as the number in the square.

Given that a disc falls within a square, the probabilities of landing within a square with the numbers 1, 2, 5 and 10 are 0.5, 0.3, 0.12 and 0.08 respectively. It is assumed that the rolls of the discs are independent.

(a) A player pays \$5 to play the game and is given n discs. Find n if the game is fair. [4]

(b) If a player is allowed to roll 3 discs for \$2, find the probability that the player will have a profit of \$10. [4]

5. **[SRJC Prelims 17]**
A random variable X has the probability distribution given by the following table.

x	2	3	4	5
$P(X = x)$	0.2	a	b	0.45

(a) Given that $E(\lvert X - 4\rvert) = \frac{11}{10}$, find the values of a and b. [3]

(b) Two independent observations of X are taken. Find the probability that one of them is 2 and the other is at most 4. [2]

6. **[SAJC Prelims 17]**
A fairground game involves trying to hit a moving target with a gunshot. A round consists of a **maximum** of 3 shots. Ten points are scored if a player hits the target. The **round** ends **immediately** if the player misses a shot. The probability that Linda hits the target in a single shot is 0.6. All shots taken are independent of one another.

   (a) Find the probability that Linda scores 30 points in a round. [2]

   The random variable X is the number of points that Linda scores in a round.

   (b) Find the probability distribution of X. [3]

   (c) Find the mean and variance of X. [4]

   (d) A game consists of 2 rounds. Find the probability that Linda scores more points in round 2 than in round 1. [2]

7. **[RI Prelims 17]**
An unbiased cubical die has the number 1 on one face, the number 2 on two faces and the number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the die twice. Adrian pays Benny \$$a$ if the total score is 2 and \$3 if the total score is 3. However, if the total score is 4, Benny pays Adrian \$2. No payment is made otherwise.

   (a) Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4]

   The random variable X represents Benny's winnings in each round.

   (b) Given that $a = 6$, find the probability distribution of X. Hence, help Benny decide if he should accept Adrian's invitation to play the game. Justify your answer. [5]

   (c) Determine the value of a for the game to be fair. [1]

8. **[NYJC Prelims 17]**
From past records, the number of days of hospitalization for an individual with minor ailment can be modelled by a discrete random variable with probability density function given by

$$P(X = x) = \begin{cases} \frac{6-x}{15}, & \text{for } x = 1, 2, 3, 4, 5, \\ 0, & \text{otherwise.} \end{cases}$$

   An insurance policy pays \$100 per day for up to 3 days of hospitalization and \$25 per day of hospitalization thereafter.

   (a) Calculate the expected payment for hospitalization for an individual under this policy. [4]

   (b) The insurance company will incur a loss if the total payout for 100 hospitalisation claims under this policy exceed \$24 000. Using a suitable approximation, estimate the probability that the insurance company will incur a loss for 100 such claims. [4]

9. **[NJC Prelims 17]**
There are three identically shaped balls, numbered from 1 to 3, in a bag. Balls are drawn one by one at random and with replacement. The random variable X is the number of draws needed for any ball to be drawn a second time. The two draws of the same ball do not need to be consecutive.
   (a) Show that $P(X = 4) = \frac{2}{9}$ and find the probability distribution of X. [3]
   (b) Show that $E(X) = \frac{26}{9}$ and find the exact value of $Var(X)$. [3]
   (c) ** The mean for forty-four independent observations of X is denoted by $\overline{X}$. Using a suitable approximation, find the probability that $\overline{X}$ exceeds 3. [3]

10. **[MJC Prelims 17]**
The probability function of X is given by

$$P(X = x) = \begin{cases} (2x-1)\theta & \text{if } x = 1, 2, 3 \\ k & \text{if } x = 4 \\ 0 & \text{otherwise} \end{cases}$$

where $0 < \theta < \frac{1}{9}$.
   (a) Show that $k = 1 - 9\theta$. Find, in terms of θ, the probability distribution of X. [2]
   (b) Find $E(X)$ in terms of θ and hence show that

$$Var(X) = 26\theta - 106\theta^2.$$

[3]
   (c) The random variable Y is related to X by the formula $Y = a + bX$, where a and b are non-zero constants. Given that $Var(Y) = \frac{1}{3}b^2$, find the value of θ. [3]

11. **[HCI Prelims 17]**
A biased tetrahedral (4-sided) die has its faces numbered '−1', '0', '2' and '3'. It is thrown onto a table and the random variable X denotes the number on the face in contact with the table. The probability distribution of X is as shown.

x	-1	0	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$

   (a) The random variable Y is defined by $X_1 + X_2$, where X_1 and X_2 are 2 independent observations of X. Show that

$$P(Y = 2) = \frac{3}{16}.$$

[2]
   (b) In a game, a player pays \$2 to throw two such tetrahedral dice simultaneously on a table. For each die, the number on the face in contact with the table is the score of a die. The player receives \$16 if the maximum of the two scores is -1, and receives \$3 if the sum of the two scores is prime. For all other cases, the player receives nothing.
   Find the player's expected gain in the game. [4]

12. **[DHS Prelims 17]**
A new game has been designed for a particular casino using two fair die. In each round of the game, a player places a bet of \$2 before proceeding to roll the two die. The players score is the sum of the results from both die. For the scores in the following table, the player keeps his bet and receives a payout as indicated.

Score	Payout
9 or 10	\$1
2 or 4	\$5
11	\$8

For any other scores, the player loses his bet.

Let X be the random variable denoting the winnings of the casino from each round of the game.

(a) Show that $E(X) = \frac{1}{12}$ and find $Var(X)$. [4]

(b) ** $\overline{X}$ is the mean winnings of the casino from n rounds of this game.
Find $P(\overline{X} > 0)$ when $n = 30$ and $n = 50\,000$.
Make a comparison of these probabilities and comment in context of the question. [3]

13. **[ACJC Prelims 17]**
Alex and his friend stand randomly in a queue with 3 other people. The random variable X is the number of people standing between Alex and his friend.

(a) Show that $P(X = 2) = 0.2$. [2]

(b) Tabulate the probability distribution of X. [2]

(c) Find $E(X)$ and $E(X - 1)^2$. Hence find $Var(X)$. [3]

Answers

1. (a) $P(X=1)=\frac{11}{36}, P(X=2)=\frac{9}{36}, P(X=3)=\frac{7}{36}, P(X=4)=\frac{5}{36}, P(X=5)=\frac{3}{36}, P(X=6)=\frac{1}{36}$.
   (b) 2.53, 1.97.
   (c) As expectation for Ivan = -0.29, Jon has a higher expected gain.
2. (a) $P(X=2)=\frac{27}{128}$.
   (b) $P(X=-4)=\frac{81}{256}, P(X=-1)=\frac{27}{64}, P(X=2)=\frac{27}{128}, P(X=5)=\frac{3}{64}, P(X=8)=\frac{1}{256}$.
   (c) Mean amount = 6, Variance amount = 27.
   (d) Expected total = $600.
   0.168.
3. (a) $k=\frac{1}{36}$.
   (b) $E(X)=\frac{161}{36}$.
   (c) 0.112.
4. (a) $n=8$.
   (b) 0.0789.
5. (a) $a=0.25, b=0.1$.
   (b) 0.18.
6. (a) 0.216.
   (b) $P(X=0)=0.4, P(X=10)=0.24, P(X=20)=0.144, P(X=30)=0.216$.
   (c) $E(X)=11.76$. $Var(X)=136.7024$.
   (d) 0.358.
7. (a) 0.134.
   (b) $E(X)=-\frac{1}{18}$ so Benny should not accept Adrian's invitation.
   (c) $a=8$.
8. (a) $213\frac{1}{3}$.
   (b) 0.00217.
9. (a) $P(X=2)=\frac{1}{3}, P(X=3)=\frac{4}{9}$.
   (b) $E(X)=\frac{26}{9}, Var(X)=\frac{44}{81}$.
   (c) 0.159.
10. (a) $P(X=1)=\theta, P(X=2)=3\theta, P(X=3)=5\theta, P(X=4)=1-9\theta$.
    (b) $E(X)=4-14\theta, Var(X)=26\theta-196\theta^2$.
    (c) $\theta=0.0144$.

11. (a) $\frac{3}{16}$.
    (b) $\$ - 0.25$.

12. (a) $Var(X) = 9.08$.
    (b) $0.560, 1.00$.
    The more rounds the game is played, the higher the chance of the casino receiving a positive average winnings. In other words, it is almost certain that the casino will win in the long run.

13. $P(X = 0) = 0.4, P(X = 1) = 0.3, P(X = 2) = 0.2, P(X = 3) = 0.1$.
    1, 1, 1.