1. **[NYJC H1 Prelims 18 (modified)]**
   A manufacturer produces balloons of which 40% are oval and 60% are round. 20 balloons are randomly selected and packed into a packet.

   (a) In a randomly selected packet of balloons, find the probability that

   i. 14 of them are round, [1]

   ii. at least half of them are round. [2]

   (b) 6 packets of balloons are randomly selected. Find the probability that less than 4 of them have at least half of the balloons that are round. [2]

   (c) ** Instead of packing 20 balloons into one packet, the manufacturer decides to pack 80 balloons into one packet. 60 packets of balloons are randomly selected. Estimate the probability that on average, at most 49 balloons are round. [3]

2. **[IJC Prelims 17]**
   At a hospital, records show that 84.5% of patients turn up for their appointments. It is known that on any day, the doctor has time to see 20 patients.

   On one particular day, there are 20 patients who make appointments to see the doctor.

   (a) State, in this context, one condition that must be met for the number of patients who turn up for their appointments to be well modelled by a binomial distribution. [1]

   For the remainder of this question, assume that the condition stated in part (a) is met.

   (b) Find the probability that more than 15 patients turn up for their appointments. [2]

   (c) Given that at least 12 patients turn up for their appointments, find the probability that more than 2 patients fail to turn up for their appointments. [3]

   (d) To improve efficiency, the hospital decides to increase the number of appointments that can be made on each day.

   Given that there will still be enough time for the doctor to see 20 patients, find the greatest number of appointments that can be made so that there is a probability of at least 0.85 of the doctor having time to see all patients who turn up. [2]

3. **[MJC Prelims 18]**
A bag contains 2 red balls, 3 yellow balls and 1 blue ball. Sue and Ben play a game where each takes turns to draw a ball from the bag, with replacement. The number of red balls obtained in n fixed draws from the bag is denoted by R.

   (a) State, in context, an assumption satisfied by R for it to be well modelled by a binomial distribution. [1]

   (b) Sue and Ben each draws n times from the bag. Find the least n such that the probability of both getting a total of at most 10 red balls is not more than 0.5. [3]

   (c) Sue and Ben each draws 5 times from the bag. The player with more red balls drawn wins. Otherwise, the game ends in a draw. Find the probability that Sue wins the game if she draws more than 3 red balls. [3]

   (d) In a variation of the game, Sue draws balls at random from the bag, one at a time without replacement, and stops when she obtains 2 yellow balls. The total number of balls Sue has to draw from the bag before she stops is denoted by T. Find $E(T)$ and $Var(T)$. [5]

4. **[TPJC Prelims 17]**
A geologist splits rocks to look for fossils. On average 7% of the rocks selected from a particular area contain fossils.

   The geologist selects a random sample of 20 rocks from this area.

   (a) Find the probability that at least three of the rocks contain fossils. [2]

   A random sample of n rocks is selected from this area.

   (b) The geologist wants to have a probability of 0.8 or greater of finding fossils in at least three of these rocks. Find the least possible value of n. [3]

   In early 2017, geologists found the fossils of *zilantophis schuberti*, a new discovered species of winged serpent. On average, the proportion of rocks that contain fossils of *zilantophis schuberti* in this area is p. It is known that the modal number of fossils of *zilantophis schuberti* in a random sample of 10 rocks is 3.

   (c) Use this information to find exactly the range of values that p can take. [4]

5. **[HCI Prelims 17 (modified)]**
In a large shipment of glass stones used for the *Go* board game, a proportion p of the glass stones is chipped. The glass stones are sold in boxes of 361 pieces each. Let X denote the number of chipped glass stones in a box.

(a) Based on this context, state two assumptions in order for X to be well modelled by a binomial distribution. [2]

In the rest of the question, assume that X follows a binomial distribution.

(b) It is known that the probability of a box containing at most 2 chipped glass stones is 0.90409. Find p. [2]

(c) A box is deemed to be of inferior quality if it contains more than 2 chipped glass stones. Find the probability that, in a batch of 20 boxes of glass stones, there are more than 5 boxes of inferior quality in the batch. [3]

(d) ** Each week, a distributor purchases 50 batches of glass stones, each batch consisting of 20 boxes of glass stones. A batch will be rejected if it contains more than 5 boxes of inferior quality.

The distributor will receive a compensation of \$10 for each rejected batch in the first 20 weeks of a year, and a compensation of \$20 for each rejected batch in the remaining weeks of the year.

Assuming that there are 52 weeks in a year, use a suitable approximation to estimate the probability that the total compensation in a year is more than \$250. [5]

6. **[AJC Prelims 17]**
A biscuit manufacturer produces both cream and chocolate biscuits. Biscuits are chosen randomly and packed into boxes of 10. The number of cream biscuits in a box is denoted by X.

(a) On average, the proportion of cream biscuits is p. Given that

$$P(X = 1 \text{ or } 2) = 0.15,$$

write own an equation for the value of p. Hence find the value(s) of p numerically. [3]

(b) It is given instead that the biscuit manufacturer produces 3 times as many cream biscuits as chocolate biscuits.

i. Find the most likely value of X. [2]

ii. A random sample of 18 boxes is taken. Find the probability that at least 3 but fewer than 7 boxes have equal numbers of cream and chocolate biscuits. [3]

(c) ** A box of biscuits is sold at \$10. The manufacturer gives a discount of \$2 per box to its premium customers. The mean and variance of the number of boxes sold per day to each type of customers (assuming independence) are as follows

	Mean	Variance
Number of boxes sold at usual price	180	64
Number of boxes sold at discounted price	840	169

Find the approximate probability that the total amount collected per month from the sales of biscuits is not less than \$255,000, assuming that there are 30 days in a month. [4]

7. [**CJC Prelims 17**]

(a) The random variable X follows a binomial distribution $B(10, p)$.

i. Given that X has two modes, $X = 4$ and $X = 5$, find the exact value of p. [2]

ii. Given instead that $P(X \leq 9) = \frac{1023}{1024}$, find the exact value of p. [2]

(b) The random variable Y follows a binomial distribution $B(500, 0.5)$.
A sample of 30 independent values of Y is recorded.

i. Find the probability that all the values recorded are less than or equal to 256. [2]

ii. ** The mean of 30 values is calculated. Estimate the probability that this sample mean is less than or equal to 256, stating clearly the approximation used. [3]

iii. Explain why the probability found in part (ii) is larger than that found in part (i). [1]

8. [**TJC Prelims 17 (modified)**]
A factory manufactures large number of pen refills. From past records, 3% of the refills are defective. A stationery store manager wishes to purchase pen refills from the factory. To decide whether to accept or reject a batch of refills, the manager designs a sampling process.

He takes a random sample of 25 refills. The batch is accepted if there is no defective refill and rejected if there are more than 2 defective refills. Otherwise, a second random sample of 25 refills is taken. The batch is then accepted if the total number of defective refills in the two samples is fewer than 4 and rejected otherwise.

(a) Find the probability of accepting a batch. [4]

(b) If a batch is accepted, find the probability that there are 2 defective refills found in the sampling process. [3]

9. [**ACJC Prelims 17**]
A procedure for accepting or rejecting a large batch of manufactured articles is such that an inspector first selects and examines a random sample of 10 articles from the batch. If the sample contains at least 2 defective articles, the batch is rejected.

It is known that the proportion of articles that are defective is 0.065. Show that the probability that a batch of articles is accepted is 0.866, correct to three significant figures. [1]

To confirm the decision, another inspector follows the same procedure with another random sample of 10 articles from the batch.

If the conclusion of both inspectors are the same, the batch will be accepted or rejected as the case may be. Otherwise, one of the inspectors will select a further random sample of 10 from the same batch to examine. The batch is then rejected if there are at least 2 defective articles. Otherwise, it is accepted.

Find

(a) the probability that a batch is eventually accepted, [3]

(b) the expected number of articles examined per batch.

(c) In order to cut labour cost, an alternative procedure is introduced. A random sample of 10 articles is taken from the batch and if the sample contains not more than 1 defective article then the batch is accepted. If the sample contains more than 2 defective articles, the batch is rejected. If the sample contains exactly 2 defective articles, a second sample of 10 articles is taken and if this contains no defective article then the batch is accepted. Otherwise, the batch is rejected.

Given that the proportion of defective articles in the batch is p, show that the probability that the batch is accepted is A where

$$A = (1 + 9p)(1 - p)^9 + 45p^2(1 - p)^{18}.$$

[2]

(d) If the probability that, out of 100 batches inspected, more than 80 of them will be accepted is 0.98, find the value of p. [3]

10. **[DHS Prelims 17]**
A sample of 5 people is chosen from a village of large population.

(a) The number of people in the sample who are underweight is denoted by X. State, in context, the assumption required for X to be well modelled by a binomial distribution. [1]

(b) On average, the proportion of people in the village who are underweight is p. It is known that the mode of X is 2. Use this information to show that $\frac{1}{3} < p < \frac{1}{2}$. [3]

1000 samples of 5 people are chosen at random from the village and the results are shown in the table below.

x	0	1	2	3	4	5
Number of groups	93	252	349	220	75	11

(c) Using the above results, find $\overline{x}$. Hence estimate the value of p. [2]

You may now use your estimate in part (c) as the value of p.

(d) Two random samples of 5 people are chosen.

Find the probability that the first sample has at least 4 people who are underweight and has more people who are underweight than the second sample. [3]

Answers

1. (a) i. 0.124.
       ii. 0.872.
   (b) 0.0307.
   (c) 0.961.
2. (a) Whether a randomly chosen patient turns up for an appointment is independent of any other patient.
   (b) 0.812.
   (c) 0.618.
   (d) 22.
3. (a) The colour of a ball chosen is independent of another.
   (b) 16.
   (c) 0.958.
   (d) 3.5, 1.05.
4. (a) 0.161.
   (b) 60.
   (c) $\frac{3}{11} < p < \frac{4}{11}$.
5. (a) The probability of a randomly chosen glass stone being chipped is constant. Whether a glass stone is chipped or not is independent of that of any other glass stones.
   (b) $p = 0.00300$.
   (c) 0.00923.
   (d) 0.953.
6. (a) $5p(1-p)^8(2+7p) = 0.15$.
       $p = 0.0162$ or $p = 0.408$.
   (b) i. 8.
       ii. 0.0843.
       iii. 0.798.
7. (a) i. $p = \frac{5}{11}$.
       ii. $p = \frac{1}{2}$.
   (b) i. 0.0000514.
       ii. 0.998.
8. (a) 0.925.
   (b) 0.209.

9. (a) 0.951.
   (b) 22.3.
   (d) 0.08.

10. (a) The weights of the 5 people chosen are independent of each other.
   The sample is randomly chosen so that the probability that each person chosen is underweight is the same.
   (c) $\overline{x} = 1.965$.
   $p = 0.393$.
   (d) 0.0758.