1. **[DHS Prelims 17]**
The students in a college are separated into two groups of comparable sizes, Group X and Group Y. The marks for their Mathematics examination are normally distributed with means and variances as shown in the following table.

	Mean	Variance
Group X	55	20
Group Y	34	25

(a) Explain why it may not be appropriate for the mark of a randomly chosen student from the college population to be modelled by a normal distribution. [1]

(b) In order to pass the examination, students from Group Y must obtain at least d marks. Find, correct to 1 decimal place, the maximum value of d if at least 60% of them pass. [3]

(c) Find the probability that the total marks of 4 students from Group Y is less than three times the mark of a student from Group X. State clearly the mean and variance of the distribution you use in your calculation. [3]

(d) ** The marks of 40 students, with 20 each randomly selected from Group X and Group Y, are used to compute a new mean mark, $\overline{M}$.
Given that $P(|\overline{M} - 44.5| < k) = 0.9545$, find the value of k. [4]

State a necessary assumption for your calculations to hold in parts (c) and (d). [1]

2. **[AJC Prelims 17]**
Males and females visiting an amusement park have heights, in centimetres, which are normally distributed with means and standard deviations as shown in the following table:

	Mean (cm)	Standard deviation (cm)
Male	165	12
Female	155	σ

It is found that 38.29% of the females have heights between 150 cm and 160 cm.

(a) Show that $\sigma = 10.0$ cm, correct to 3 significant figures. [2]

(b) Find the probability that the height of a randomly chosen female is within 20 cm of three-quarter the height of a randomly chosen male. State an assumption that is necessary for the calculation to be valid. [4]

The amount, \$$X$, a visitor has to pay for a popular ride in the park is \$10 if the visitors height is at least 120 cm but less than 150 cm, and \$$m$ if the visitors height is 150 cm and above. If the visitors height is less than 120 cm, he/she does not need to pay for the ride.

(c) Assuming that a visitor purchasing a ticket for the ride is equally likely to be a male or female, find in terms of m, the probability distribution of X.
Given that the expected amount a visitor will pay for a ride is \$17.93, show that $m = 20.00$, correct to 2 decimal places. [4]

[Question continued on next page]

(d) Three visitors were randomly chosen. Find the probability that the total amount they paid for a ride together is more than \$40. [3]

3. **[CJC Prelims 17]**
A trading card game has rectangular cards of nominal size 64 mm wide and 89 mm long. However, due to the limited precision of the machine used to cut the cards to size, the widths of the trading cards follow a normal distribution with mean 64 mm and standard deviation 0.3 mm. The lengths of the trading cards follow an independent normal distribution with mean 89 mm and standard deviation 0.45 mm. The perimeter of the trading cards is twice the sum of its length and width.

(a) Trading cards with length 90 mm and above are called "tall" cards. Find the percentage of trading cards that are "tall". [1]

(b) Write down the distribution of the perimeter of the trading cards, in mm, and find the perimeter that is exceeded by 8% of the trading cards. [4]

A brand of rectangular card sleeves are manufactured for the trading cards and the widths of the card sleeves follow a normal distribution with mean 66 mm and standard deviation 0.45 mm, whereas the lengths of the card sleeves follow an independent normal distribution with mean 91 mm and standard deviation 0.675 mm.

For a card sleeve to fit the trading card nicely, the dimensions of the sleeves must be larger than the dimensions of the trading card, but there should only be a maximum allowance of 1.2 mm on each side.

(c) Find the probability that a randomly chosen card sleeve fits a randomly chosen trading card nicely, stating clearly the parameters of any distribution used. [5]

4. **[IJC Prelims 17]**
In order to recruit the best possible employees, a large corporation has designed an entrance test that consists of three components, namely Logical Reasoning, Personality and Communication. The scores obtained by candidates in each of the three components are independent random variables L, P and C which are normally distributed with means and standard deviations as shown in the table.

	Mean	Standard deviation
Logical Reasoning, L	35.2	5.2
Personality, P	24.6	3.8
Communication, C	29.3	4.3

(a) For a particular role in the corporation, the Logical Reasoning and the Personality scores of a candidate is valued and hence a special score of $3L + 2P$ is computed.

i. Find the special score that is exceeded by only 1% of the candidates taking the test. Leave your answer in 1 decimal place. [4]

ii. Five candidates are selected randomly. Find the probability that three of them obtained a special score of more than 150, and the other two obtained less than 140. [3]

(b) For another role in the corporation, a candidate must achieve a result such that his special score of $3L+2P$ differs from $5C$ by less than 25. Find the percentage of candidates who will be able to achieve this. [4]

5. **[TPJC Prelims 17]**
A manufacturing plant processes raw material for a supplier. An order placed with the plant is considered to be a bulk order when a worker is expected to process more than 300 kg (kilograms) of raw material.

Albert uses a machine to process X kg of raw material and Bob uses a separate machine to process Y kg of raw material on a working day. X and Y are independent random variables with the distributions $N(296, 8^2)$ and $N(290, 12^2)$ respectively.

(a) Find the probability that Albert processes more than 300 kg of raw material on a randomly selected working day. [2]

(b) Find the probability that, over a period of 15 independent working days, there are exactly four working days on which Albert processes more than 300 kg of raw material. [2]

(c) Find the probability that the total amount of raw material Bob processes over two working days exceeds twice the amount of raw material Albert processes on one working day. [4]

The plant receives a bulk order and Albert wants to have a probability of at least 0.95 of meeting the order.

(d) This can be done by changing the value of μ, the mean amount of raw material Albert processes using the machine, but the standard deviation remains unchanged. Find the least value of μ. [3]

6. **[JJC Prelims 17]**
In this question you should state clearly the values of the parameters of any normal distributions you use.

The mass of a tomato of variety A has normal distribution with mean 80 g and standard deviation 11 g.

(a) Two tomatoes of variety A are randomly chosen. Find the probability that one of the tomatoes has mass more than 90 g and the other has mass less than 90 g. [3]

The mass of a tomato of variety B has normal distribution with mean 70 g. These tomatoes are packed in sixes using packaging that weights 15 g.

(b) The probability that a randomly chosen pack of 6 tomatoes of variety B including packaging, weighs less than 450 g is 0.8463. Show that the standard deviation of the mass of a tomato of variety B is 6 g, correct to the nearest gram. [4]

(c) Tomatoes of variety A are packed in fives using packaging that weighs 25 g. Find the probability that the total mass of a randomly chosen pack of variety A is greater than the total mass of a randomly chosen pack of variety B, using 6 g as the standard deviation of the mass of a tomato of variety B. [5]

7. **[NJC 17 Prelims]**
The number of days of gestation for a Dutch Belted cow is normally distributed, with a mean of μ days and a standard deviation of σ days. 8.08% of this cattle breed has a gestation period shorter than 278 days whereas 21.2% has a gestation period longer than 289 days. Find the values of μ and σ, giving your answers correct to 3 significant figures. [3]

   (a) ** Find the probability that the mean gestation period for thirty-two randomly chosen Dutch Belted cows is more than 287 days. State a necessary assumption for your calculation to be valid. [3]

   For another cattle breed, the Jersey cow, the number of days of gestation is normally distributed with a mean of 278 days and a standard deviation of 2.5 days. During gestation, a randomly chosen pregnant Dutch Belted cow eats 29 kg of feed daily while a randomly chosen pregnant Jersey cow eats 26 kg of feed daily.

   (b) Find the value of a such that during their respective gestation periods, there is a probability of 0.35 that the amount of feed consumed by a randomly chosen pregnant Jersey cow exceeds half of the amount consumed by a randomly chosen pregnant Dutch Belted cow by less than a kg. Express your answer to the nearest kg. [2]

   (c) Calculate the probability that during their respective gestation periods, the difference between the amount of feed consumed by three randomly chosen pregnant Dutch Belted cows and four randomly chosen pregnant Jersey cows is more than 4000 kg. State clearly the parameters of the distribution used in the calculation. [3]

8. **[RVHS Prelims 17]**
Each month the amount of electricity, X measured in kilowatt-hours (kWh), used by a household in a particular city may be assumed to follow a normal distribution with mean 950 and standard deviation σ. The charge for electricity used per month is fixed at $0.22 per kWh.

   (a) Given that 65% of the households uses less than 960 kWh of electricity in a month, find the value of σ, correct to 1 decimal place. [2]

   For the rest of the question, σ is the value found in part (a).

   (b) Find the probability that the difference in the amount of electricity used among 2 randomly chosen households in a particular month is not more than 30 kWh. [3]

   (c) In the month of August, the mayor of the city decides to provide 50% and 30% subsidies for the electricity bills of households in the North and South districts of the city respectively. Find the probability that the total electricity bill of 2 randomly chosen North district households and 1 South district household is less than $360. [4]

   (d) In December, a random sample of n households is chosen to study the mean monthly electricity usage per household in the city. Find the least value of n if the probability of the sample mean being less than 955 kWh is at least 0.9. [3]

9. **[SAJC Prelims 17]**
A factory manufactures round tables in two sizes: small and large. The radius of a small table, measured in cm, has distribution $N(30, 2^2)$ and the radius of a large table, measured in cm, has distribution $N(50, 5^2)$.

(a) Find the probability that the sum of the radius of 5 randomly chosen small tables is less than 160 cm. [2]

(b) Find the probability that the sum of the radius of 3 randomly chosen small tables is less than twice the radius of a randomly chosen large table. [1]

(c) State an assumption needed in your calculation in part (b). [1]

A shipment of 12 large tables is to be exported. Before shipping, a check is done and the shipment will be rejected if there are at least two tables whose radius is less than 40 cm.

(d) Find the probability that the shipment is rejected. [3]

The factory decides now to manufacture medium sized tables. The radius of a medium sized table, measured in cm, has distribution $N(\mu, \sigma^2)$. It is known that 20% of the medium sized tables have radius greater than 44 cm and 30% have radius of less than 40 cm.

(e) Find the values of μ and σ. [4]

Answers

1. (a) The distribution may become bimodal when the data for both groups are combined.
   (b) 32.7.
   (c) 0.958.
   (d) $k = 1.50$.
   The marks of the students are independent of one another.
2. (b) 0.201.
   $P(X = 0) = 0.00016056, P(X = 10) = 0.20693,$
   $P(X = m) = 0.79291.$
   (c) 0.889.
3. (a) 1.31%.
   (b) $t = 307.51$ mm.
   (c) 0.525.
4. (a) i. 195.2.
   ii. 0.0875.
   (b) 61.3%.
5. (a) 0.309.
   (b) 0.214.
   (c) 0.303.
   (d) 314.
6. (a) 0.297.
   (c) 0.364.
7. (a) 0.0119.
   (b) 2058.
   (c) 0.660.
8. (a) 26.0.
   (b) 0.585.
   (c) 0.796.
   (d) 45.
9. (a) 0.987.
   (b) 0.828.
   (d) 0.0294.
   (e) $\mu = 41.5, \sigma = 2.93$.