SRJC 2015 H1 Maths Prelim Paper

Section A: Pure Mathematics [35 marks]

1 Differentiate the following with respect to x.

(i) $(2x^5 + 5)^3$ [1]

(ii) e^{4-x^2} [2]

2 **(i)** Write the $\dfrac{2+6x}{2x-1}$ in the form $a+\dfrac{b}{2x-1}$, where a and b are constants to be determined. [1]

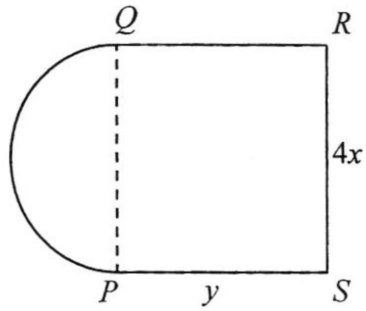
(ii) Sketch the graph of $y=\dfrac{2+6x}{2x-1}$, stating clearly all coordinates of axial intercepts and equations of asymptotes. [3]

(iii) Hence, solve the inequality $\dfrac{5}{2x-1} \geq -3$. [2]

3 **(a)** The volume of an inflated spherical balloon is leaking and the surface area of the spherical balloon decreases at a steady rate of 12 units ^{2} s $^{-1}$. Find the exact rate of change of the radius at the instant when the volume of balloon reaches $\dfrac{9}{16}\pi$ units^3. [3]

[Volume of sphere = $\dfrac{4}{3}\pi r^3$, surface area of sphere = $4\pi r^2$]

(b)

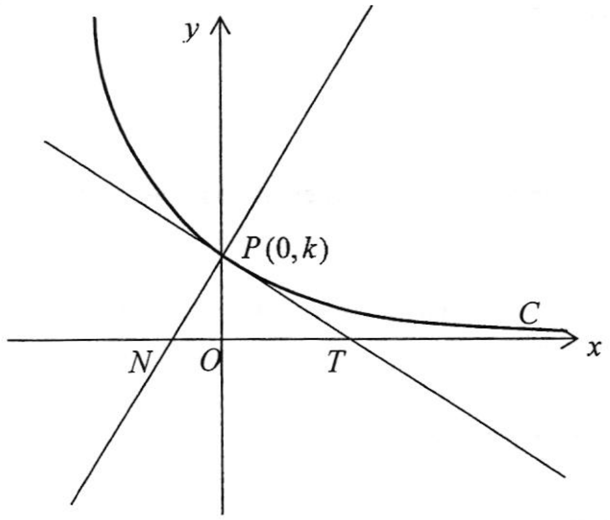


A flower-bed has the shape of a semi-circle joined to a rectangle as shown in the diagram. It is given that $PS = y$ m and $RS = 4x$ m. The flower-bed has a fixed area of 20 m^2. The cost of fencing the straight parts is \$10 per metre and the cost of fencing the semi-circular part is \$20 per metre.

(i) Show that the cost, \$$C$, of fencing the perimeter of the flower-bed is given by $C=(40+30\pi)x+\dfrac{100}{x}$. [4]

(ii) Using differentiation, find the minimum cost of fencing the flower-bed, giving your answer correct to 2 decimal places. [5]

4 The diagram below shows part of the curve C with equation $y = \dfrac{k}{3x+1}$, where k is a positive constant.



The curve cuts the y-axis at the point P (0, k). The tangent at P cuts the x-axis at T and the normal at P cuts the x-axis at N as shown above.

(i) Show that the equation of tangent at P is $y = -3kx + k$ and find the equation of normal at P in terms of k. [3]

(ii) Show that the x-coordinate of N is $-3k^2$ and find the x-coordinate of T. [2]

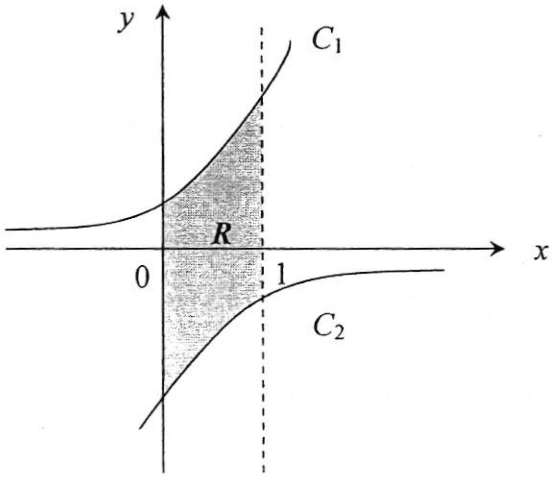
(iii) Show that the area of triangle PTN is $\dfrac{1}{6}k\left(1+9k^2\right)$. [2]

5 **(a)** If $y = \ln(2x^2+1)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Hence evaluate $\displaystyle\int \frac{3x}{4x^2+2}\,\mathrm{d}x$. [3]

(b) The diagram shows the graphs of

$$C_1 : y = \mathrm{e}^{3x} + 2k \qquad \text{and} \qquad C_2 : y = 1 - \mathrm{e}^{-2x} - k,$$

where k is a positive constant.



The region R is bounded by the lines $x = 0$ and $x = 1$, and the curves C_1 and C_2. Find the exact area of the region R in terms of k. [4]

Section B: Statistics [60 marks]

6 **(a)** A junior college in Singapore has 20 year-one classes and 20 year-two classes. Each year-one class has 20 students and each year-two class has 25 students. A college survey is implemented to determine the average amount a student spends on lunch. A group of students proposes the following approach to select a sample of 160 students.

"Randomly choose 2 boys and 2 girls in each of the 40 classes to answer the survey."

It may be assumed that there are at least 2 boys and 2 girls in each class.

(i) Explain, with a suitable reason, why the proposed approach described above will not give rise to a random sample. [1]

(ii) Describe an alternative sampling method which would give a random sample with the same sample size. [3]

(b) In an examination graded out of 100, the mean mark obtained by a randomly chosen student is 80.3 marks and the standard deviation is 10.3 marks. Explain with a suitable reason if it is reasonable to assume that the mark obtained by a randomly chosen student in the examination is normally distributed. [1]

The results of 40 randomly chosen students are recorded. Estimate the probability that the average mark of the 40 students is at most 78. [2]

7 A grocer sells eggs in cartons of 36 eggs. On average, 4 out of 5 eggs have mass more than 10 grams. The masses of the eggs are independent of one another.

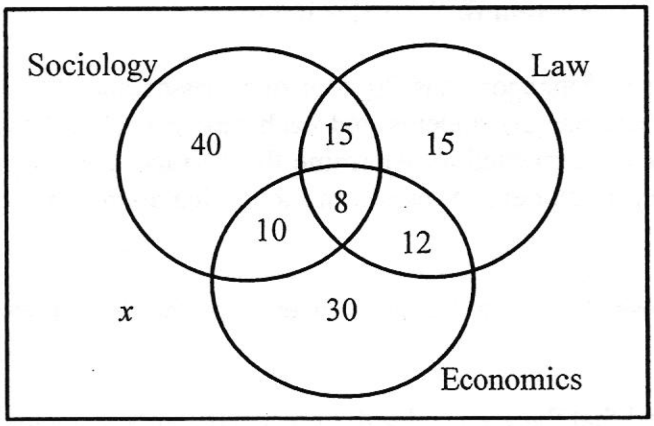
(i) Let X be the number of eggs with mass more than 10 grams in a carton. State, in context, an assumption needed for X to be well modelled by a binomial distribution. [1]

A quality control test is passed if there are at least 24 but less than 30 eggs in a carton having a mass of more than 10 grams.

(ii) Find the probability that a randomly chosen carton will pass the quality control test. [2]

A customer buys a batch of 60 cartons of eggs from the grocer.

(iii) Using a suitable approximation, estimate the probability that not more than 50% of the cartons in the batch will pass the quality test. State the mean and variance of the distribution that you use. [4]

8 

A group of students who applied to a university was asked whether they applied to the law course, the sociology course and the economics course. The numbers applying to different combination of courses are shown in the Venn diagram. The number of students who did not apply to any of these three courses is x. One student is chosen at random.

L is the event that the student applied to the law course.

S is the event that the student applied to the sociology course.

E is the event that the student applied to the economics course.

(i) Write down the expressions for P(L) and P(E) in terms of x. Given that L and E are independent, show that $x = 20$. [3]

Using this value of x, find

(ii) $P(E' \cap S)$ [1]

(iii) $P(S \mid L)$. [2]

9 In an investigation on how the duration of training (t years) affects the performance in swimming (y seconds) for a particular style, the following data are tabulated.

t (years)	9.2	9.6	10.1	10.5	11.1	11.4	12.1
y (seconds)	27.8	27.1	26.8	26.0	25.4	23.8	26.5

(i) Sketch a scatter diagram for the data, as shown on your calculator. [2]

(ii) Identify one data point which should be removed. [1]

(iii) Ignoring this erroneous data point, find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iv) Find the regression line of y on t and use it to estimate y for $t = 14$. Comment on the reliability of your estimation. [3]

It is found that the performance in swimming was wrongly recorded at the erroneous point and the correct performance in swimming at that point is now rectified. With the corrected set of data, the equation of regression line of y on t is found to be

$$y = 39.801 - 1.3136t$$

(v) Determine the correct value of the performance in swimming at the previously erroneous point. [3]

10 A factory produces fabric A which has a mean breaking strength of not more than 56.5 lbs. A strengthening fibre is woven into fabric A. The breaking strength, x lbs, of a random sample of 75 pieces of the new composite fabric is summarized by

$$\Sigma x = 4365.8 \quad \text{and} \quad \Sigma(x - \bar{x})^2 = 3021.87$$

It is assumed that the breaking strength of fabric A follows a normal distribution.

(i) Calculate the unbiased estimate of population mean and variance. [2]

(ii) Test, at 2% level of significance, whether the introduction of new fibre improves the breaking strength of the fabric. [4]

Another factory produces fabric B which has a mean breaking strength of m and variance 35.267. A newly discovered fibre is interwoven into the fabric B. The breaking strength, y, of 30 experimental pieces of new fabric has mean 78.12.

A test at α% level of significance on the effect on the breaking strength of the new fabric is conducted.

(iii) **(a)** Suppose that the test concluded at 8% level of significance that the breaking strength is not the same as before. Determine the range m. [4]

(b) Suppose that m = 80 and the test concluded that there is no effect on the breaking strength. Find the range of α. [2]

11 **(a)** The time taken, x, for a random student from a school to complete a given list of mathematics problems follows a normal distribution with mean μ minutes and standard deviation σ minutes. Given that 40% of students in the school take more than 1 hour to complete the problems and one out of five of students can complete in less than 40 minutes, find the mean and variance. [4]

(b) A man's exercise regime consists of brisk walking and cycling. The time he spends on brisk walking and the time he spends on cycling follow normal distributions with means and standard deviations as shown below.

	Mean (in minutes)	Standard deviation (in minutes)
Brisk walking	22	4
Cycling	45	12

(i) In one particular week, the man did brisk walking on 4 of the days. Find the probability that he did brisk walking for more than 25 minutes on exactly 3 days and less than 20 minutes on exactly 1 day. [3]

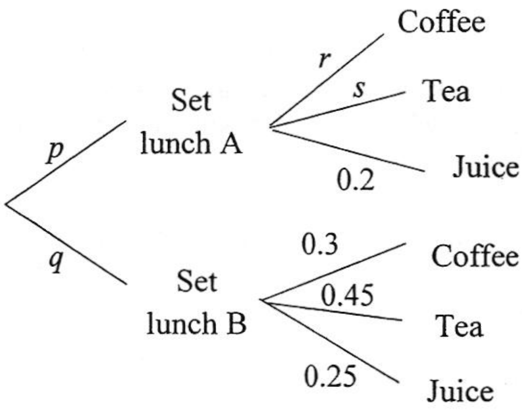
It is assumed that each time, the man maintained a constant brisk walking pace of 0.2 kilometres per minute and a constant cycling speed of 0.45 kilometres per minute. [Note: distance = speed × time]

(ii) In a self-challenge, the man aim to cover a minimum of 400 kilometres through brisk walking and cycling within 4 weeks. He planned a 4-weeks program to do 1 day of brisk walking and 4 days of cycling per week.

Find the probability that the goal is met at the end of the 4-weeks program. [3]

(iii) State one assumption necessary in the calculation in part **(ii)**. [1]

12 A restaurant has a fixed lunch menu. Customers may order either only set lunch A or set lunch B. With each set lunch, they may select one drink of either coffee, tea or juice. A tree diagram indicating the probabilities for each combination of set lunch and drink is as shown below.



It is known that twice as many customers order set lunch A as set lunch B and 32% of the customers order set lunch A with coffee.

(i) Find the values of p, q, r and s. [3]

A customer at the restaurant is chosen at random.

(ii) Find the probability that the customer orders coffee. [1]

(iii) If the customer chooses coffee, what is the probability that the customer ordered set lunch A? [2]

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