§3. Discrete Random Variables and Binomial Distribution: Summary Notes

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2.1 Discrete Random Variables

- A **discrete random variable** is a random variable that takes only a countable number of values.
- $\boxed{\sum P(X=x)=1}$ for all discrete random variables X.
- The **probability distribution** of a discrete r.v. can be summarized in a table or a function.
- The mean, or **expectation** of a discrete r.v. X is given by $\boxed{\mu = \mathrm{E}(X) = \sum xP(X=x).}$
- $\mathrm{E}(X^2) = \sum x^2 P(X=x)$.
- The **variance** or a discrete r.v. X is given by $\boxed{\sigma^2 = \mathrm{Var}(X) = \mathrm{E}(X^2) - \big(\mathrm{E}(X)\big)^2.}$
- $\mathrm{E}(aX \pm bY) = a\mathrm{E}(X) \pm b\mathrm{E}(Y)$.
- $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$.
- $\mathrm{E}(X_1 + \cdots + X_n) = n\mathrm{E}(X)$.
- $\mathrm{E}(nX) = n\mathrm{E}(X)$.
- $\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$.
- $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$.
- $\mathrm{Var}(X_1 + \cdots + X_n) = n\mathrm{Var}(X)$.
- $\mathrm{Var}(nX) = n^2\mathrm{Var}(X)$.

Remark: The variance formulas assume **independence** between X and Y.

2.2 Binomial Basics.

- The conditions for a random variable to be modeled by a **binomial distribution** are:
  - There are n fixed trials.
  - For each trial, there is a notion of "success" with probability p. The outcome of each trial must be either "success" or "failure".
  - p is the same for each trail.
  - The outcomes of each trial are independent.
- In that case, we write $X \sim B(n, p)$.
- Some formulas associated with a binomial distribution (in MF15):
  - $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$.
  - The mean of X, $\mathrm{E}(X) = np$.
  - The variance of X, $\mathrm{Var}(X) = np(1-p)$.
- Sometimes the probability of failure is denoted by $q = 1 - p$, the mean by μ and the variance by σ^2. The standard deviation is denoted by σ.

2.3 Using the GC: binompdf and binomcdf.

- The GC function BINOMPDF gives us the probability of $P(X = x)$.
- The GC function BINOMCDF gives us the probability of $P(X \leq x)$.
- For all other inequality relations, we can always convert to the cdf form by appropriate manipulation. For example,
  - $P(X < 5) =$
  - $P(2 \leq X < 7) =$
  - $P(X \geq 12) =$
  - $P(X > 6) =$

2.4 Using the GC: table and graphs.

- The BINOMPDF and BINOMCDF functions allow us to calculate probabilities when n, p and x are known.
- Sometimes, a probability will be given and we have to work backwards to obtain n, p or x.
- For some questions, we will be guided to make use of the binomial distribution formula and some algebra to solve for our unknowns. Otherwise,
  - n and x are integers so a **table method** can be used.
  - p is a real number so a **graphical method** needs