1. **[HCI Prelims 13]**
Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point C is on OA is such that $OC : CA = 1 : 2$, the point D on OB is such that $OD : DB = 3 : 4$ and the point M on CD is such that $11CM = 5CD$.

   (a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

   (b) By considering a cross product, find the ratio of the area of triangle OCM to the area of triangle OAB. [3]

2. **[MI 13 Prelims (modified)]**

   (a) Given that $\cos\theta = \frac{1}{3}$, where θ is the angle between the vectors $\mathbf{i} + \mathbf{j} + \lambda\mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, find the value of the constant λ. [3]

   (b) Referred to the origin O, the position vectors of the points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. OAB is a triangle. Show that area of the triangle OAB cannot exceed $\frac{1}{2}|\mathbf{a}||\mathbf{b}|$. [2]

3. **[HCI Prelims 14]**
Relative to the origin O, the points A, B and C have position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{a} + \mathbf{b}$ respectively.

   The point X is on AB produced such that $AB : AX$ is $1 : 5$ and the point Y is such that $ACXY$ is a parallelogram.

   It is given that the area of the triangle OAB is 1 square unit and $\mathbf{b}$ is a unit vector.

   (a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of X and Y. Hence show that $OABY$ is a trapezium. [5]

   (b) Give a geometrical meaning of $|(\mathbf{a} + \mathbf{b}) \cdot \mathbf{b}|$. [1]

   (c) Find the area of $ACXY$. Hence find the shortest distance from X to the line that passes through A and C. [3]

4. **[JJC Prelims 14]**
Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively.

   It is given that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other and have the same magnitude of 3 units each.

   Given that A, B and C are collinear,

   (a) Show that $\mathbf{c}$ can be expressed as $\mathbf{c} = k\mathbf{b} + (1 - k)\mathbf{a}$, where k is a constant. [1]

   (b) Find $|\mathbf{a} \times \mathbf{c}|$ in terms of k, and state its geometrical meaning. [4]

   (c) It is given that the area of triangle OAC is three times the area of triangle OAB. Find the two possible values of k.
   Given also that the length of projection of OC onto OA is 12 units, find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

5. **[IJC Prelims 13]**
Referred to an origin O, the position vectors of two points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively.

(a) State the geometric interpretation of $|\mathbf{a} \times \mathbf{b}|$. [1]

(b) Given that $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3$, find the value of $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$. [2]

(c) Given further that the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$, find the exact value of $|(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})|$. [4]

6. **[RI Prelims 13]**
$OABC$ is a trapezium such that OA is parallel to CB, and $CB : OA = k : 1$, where K is a positive constant and $k \neq 1$.

Given that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, and X and Y are midpoints of OB and AC respectively, find the following vectors in terms of $k, \mathbf{a}$ and $\mathbf{b}$:

(a) $\overrightarrow{OC}$, [1]

(b) $\overrightarrow{OY}$. [2]

(c) Hence show that XY is parallel to OA. [2]

(d) It is given that OB and AC intersect at the point D. Find the ratio, in terms of k, between the area of the triangle XYD and the area of the triangle BCD. [2]

7. **[RVHS Prelims 13]**
Referred to an origin O, the position vectors of two points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(a) The point C lies on AB produced such that $AB : BC$ is $1 : 3$. Find the position vector of C. [2]

(b) The point D lies on OB produced such that $OB : OD$ is $1 : k$. Given that CD is perpendicular to AB,
show that $k = \dfrac{4|\mathbf{b}|^2 - 7\mathbf{a} \cdot \mathbf{b} + 3|\mathbf{a}|^2}{|\mathbf{b}|^2 - \mathbf{a} \cdot \mathbf{b}}$. [3]

(c) Show that $OADC$ cannot be a parallelogram. [2]

(d) Find the area of triangle ABD in terms of k. [3]

8. **[SAJC Prelims 13]**
With respect to the origin O, the position vectors of the points A and B are

$$-3\mathbf{i} + 2\mathbf{j} + m\mathbf{k} \quad \text{and} \quad -12\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$$

respectively, where $m < 0$. The point P is on line AB such that $AP : PB = 2 : 1$.

(a) Find the coordinates of the point P in terms of m. [2]

(b) It is given that the area of triangle OAP is $4\sqrt{13}$ units^2. Show that $m = -2$. [4]

(c) Hence or otherwise find the shortest distance from O to the line AP. [3]

9. **[YJC Prelims 18]**
Relative to the origin O, the position vectors of points A, B and N are $\mathbf{a}, \mathbf{b}$ and $\mathbf{n}$ respectively. It is given that N lies on AB, between A and B, such that $\mathbf{a} \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n}$ and $\angle AON$ and $\angle BON$ are $30°$ and $60°$ respectively.

   (a) Show that AB is perpendicular to ON. [2]

   (b) State the geometrical meaning of $|\mathbf{a} \times \mathbf{n}|$. [1]

   (c) State the geometrical meaning of $\left|\mathbf{a} \times \frac{\mathbf{n}}{|\mathbf{n}|}\right|$ and show that its values is $k|\mathbf{n}|$, where k is an exact value to be found. [2]

   (d) Find the ratio $AN : NB$ and express $\mathbf{n}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

10. **[TPJC Prelims 18]**
Referred to the origin O, points A and B have position vectors given by

$$\mathbf{a} = -p\mathbf{i} + 2p\mathbf{j} + 2p\mathbf{k} \quad \text{and} \quad \mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$$

respectively, where $p > 0$.

   (a) Given that $\mathbf{a}$ is a unit vector, find the exact value of p. [2]

   (b) Give a geometrical interpretation of $|\mathbf{a} \cdot \mathbf{b}|$. [1]

   Point C lies on AB, between A and B, such that $AC : CB = 3 : 2$.

   (c) Find the position vector of C. [2]

   (d) Find the exact area of triangle OBC. [3]

11. **[NYJC Prelims 18]**
Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors.

   (a) By using scalar product, show that the vector $\mathbf{a} + \mathbf{b}$ is the bisector of angle AOB. [3]

   (b) If the area of the triangle AOB is $\frac{1}{\sqrt{10}}$ units^2, state the exact value of the sine of angle AOB. [1]

12. **[NYJC Prelims 18]**
Referred to the origin O, points C and D have position vectors $7\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$ and $4\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ respectively.

   (a) Using vector product, find the exact shortest distance of the line, passing through points C and D, from the origin [4]

   (b) Find angle OCD. [2]

13. **[EJC Prelims 18]**
** Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$. Point C is on the line which contains A and is parallel to $\mathbf{b}$. It is given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are both of magnitude 2 units and are at an angle of $\sin^{-1}\frac{1}{6}$ to each other. If the area of triangle OAC is 3 units^2, use vector product to find the possible position vectors of C in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

14. **[EJC Prelims 18]**

Referred to the origin O, the points P and Q have position vectors $\mathbf{p}$ and $\mathbf{q}$ where $\mathbf{p}$ and $\mathbf{q}$ are non-parallel, non-zero vectors. Point R is on PQ produced such that $PQ : QR = 1 : \lambda$. Point M is the mid-point of OR.

(a) Find the position vector of R in terms of $\lambda, \mathbf{p}$ and $\mathbf{q}$. [1]

F is on OQ such that F, P and M are collinear.

(b) Find the ratio $OF : FQ$, in terms of λ. [4]

Answers

1. (a) $\frac{2}{11}\mathbf{a} + \frac{15}{77}\mathbf{b}$.

   (b) $\frac{5}{77}$.

2. $\frac{7}{4}$.

3. (a) $\overrightarrow{OX} = 5\mathbf{b} - 4\mathbf{a}$.
   $\overrightarrow{OY} = 4\mathbf{b} - 4\mathbf{a}$.

   (b) It is the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$.

   (c) Area = 10 units square.
   Shortest distance = 10.

4. (b) $9|k|$.
   It is the area of a parallelogram formed with sides OA and OC.

   (c) $k = \pm 3$.
   $\mathbf{c}$ = -3$\mathbf{b}$ + 4$\mathbf{a}$

5. -5.
   $6\sqrt{3}$.

6. (a) $\mathbf{b} - k\mathbf{a}$.

   (b) $\frac{1-k}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$.

   $\dfrac{(1-k)^2}{4k^2}$.

7. (a) $\mathbf{c} = 4\mathbf{b} - 3\mathbf{a}$.

   (d) $|1-k||\mathbf{a} \times \mathbf{b}|$.

8. (a) $\left(-9, 6, \dfrac{8+m}{3}\right)$.

   (c) $4\sqrt{\dfrac{13}{17}}$.

9. (b) Area of the parallelogram with sides OA and ON.

   (c) Perpendicular distance from A to ON.
   $k = \frac{1}{\sqrt{3}}$.

   (d) 1:3.
   $\mathbf{n} = \frac{3\mathbf{a}+\mathbf{b}}{4}$.

10. (a) $p = \frac{1}{3}$.

    (b) It is the length of projection of OB on OA.

    (c) $\frac{1}{15}(34\mathbf{i} + 31\mathbf{j} + 4\mathbf{k})$.

    (d) $\frac{\sqrt{221}}{15}$.

11. $\frac{2}{\sqrt{10}}$.

12. (a) $3\sqrt{6}$.
    (b) $35.3°$.

13. $\overrightarrow{OC} = \mathbf{a} \pm 9\mathbf{b}$.

14. (a) $(1+\lambda)\mathbf{q} - \lambda\mathbf{p}$.
    (b) $1+\lambda : 1$.