

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2007

3 hours

Additional Materials: Answer Paper
 Graph paper
 List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

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At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

1 Show that $\frac{2x^2 - x - 19}{x^2 + 3x + 2} - 1 = \frac{x^2 - 4x - 21}{x^2 + 3x + 2}$. [1]

Hence, without using a calculator, solve the inequality

$$\frac{2x^2 - x - 19}{x^2 + 3x + 2} > 1. \quad [4]$$

2 Functions f and g are defined by

$$\begin{aligned} f : x &\mapsto \frac{1}{x-3} && \text{for } x \in \mathbb{R}, x \neq 3, \\ g : x &\mapsto x^2 && \text{for } x \in \mathbb{R}. \end{aligned}$$

(i) Only one of the composite functions fg and gf exists. Give a definition (including the domain) of the composite that exists, and explain why the other composite does not exist. [3]

(ii) Find $f^{-1}(x)$ and state the domain of f^{-1} . [3]

3 (a) Sketch, on an Argand diagram, the locus of points representing the complex number z such that $|z + 2 - 3i| = \sqrt{13}$. [3]

(b) The complex number w is such that $ww^* + 2w = 3 + 4i$, where w^* is the complex conjugate of w . Find w in the form $a + ib$, where a and b are real. [4]

4 The current I in an electric circuit at time t satisfies the differential equation

$$4 \frac{dI}{dt} = 2 - 3I.$$

Find I in terms of t , given that $I = 2$ when $t = 0$. [6]

State what happens to the current in this circuit for large values of t . [1]

5 Show that the equation $y = \frac{2x+7}{x+2}$ can be written as $y = A + \frac{B}{x+2}$, where A and B are constants to be found. Hence state a sequence of transformations which transform the graph of $y = \frac{1}{x}$ to the graph of $y = \frac{2x+7}{x+2}$. [4]

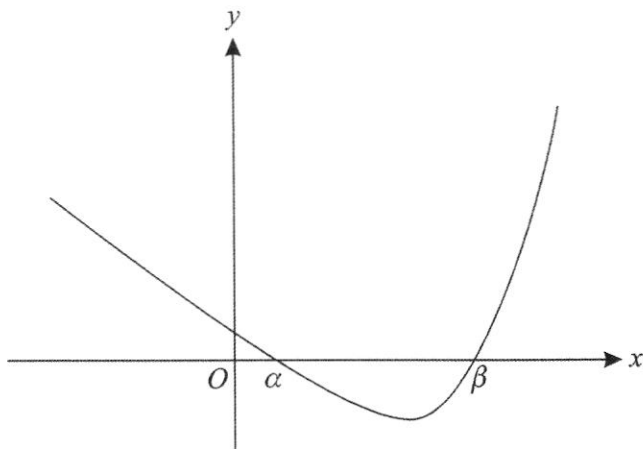
Sketch the graph of $y = \frac{2x+7}{x+2}$, giving the equations of any asymptotes and the coordinates of any points of intersection with the x - and y -axes. [3]

- 6 Referred to the origin O , the position vectors of the points A and B are

$$\mathbf{i} - \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$$

respectively.

- (i) Show that OA is perpendicular to OB . [2]
- (ii) Find the position vector of the point M on the line segment AB such that $AM : MB = 1 : 2$. [3]
- (iii) The point C has position vector $-4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Use a vector product to find the exact area of triangle OAC . [4]
- 7 The polynomial $P(z)$ has real coefficients. The equation $P(z) = 0$ has a root $re^{i\theta}$, where $r > 0$ and $0 < \theta < \pi$.
- (i) Write down a second root in terms of r and θ , and hence show that a quadratic factor of $P(z)$ is $z^2 - 2rz \cos \theta + r^2$. [3]
- ⊕ (ii) Solve the equation $z^6 = -64$, expressing the solutions in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]
- ⊕ (iii) Hence, or otherwise, express $z^6 + 64$ as the product of three quadratic factors with real coefficients, giving each factor in non-trigonometrical form. [3]
- 8 The line l passes through the points A and B with coordinates $(1, 2, 4)$ and $(-2, 3, 1)$ respectively. The plane p has equation $3x - y + 2z = 17$. Find
- (i) the coordinates of the point of intersection of l and p , [5]
- (ii) the acute angle between l and p , [3]
- (iii) the perpendicular distance from A to p . [3]



The diagram shows the graph of $y = e^x - 3x$. The two roots of the equation $e^x - 3x = 0$ are denoted by α and β , where $\alpha < \beta$.

- (i) Find the values of α and β , each correct to 3 decimal places. [2]

A sequence of real numbers x_1, x_2, x_3, \dots satisfies the recurrence relation

$$x_{n+1} = \frac{1}{3}e^{x_n}$$

for $n \geq 1$.

- (ii) Prove algebraically that, if the sequence converges, then it converges to either α or β . [2]
- (iii) Use a calculator to determine the behaviour of the sequence for each of the cases $x_1 = 0$, $x_1 = 1$, $x_1 = 2$. [3]
- (iv) By considering $x_{n+1} - x_n$, prove that

$$\begin{aligned} x_{n+1} &< x_n \text{ if } \alpha < x_n < \beta, \\ x_{n+1} &> x_n \text{ if } x_n < \alpha \text{ or } x_n > \beta. \end{aligned} \quad [2]$$

- (v) State briefly how the results in part (iv) relate to the behaviours determined in part (iii). [2]

- 10 A geometric series has common ratio r , and an arithmetic series has first term a and common difference d , where a and d are non-zero. The first three terms of the geometric series are equal to the first, fourth and sixth terms respectively of the arithmetic series.

- (i) Show that $3r^2 - 5r + 2 = 0$. [4]
- (ii) Deduce that the geometric series is convergent and find, in terms of a , the sum to infinity. [5]
- (iii) The sum of the first n terms of the arithmetic series is denoted by S . Given that $a > 0$, find the set of possible values of n for which S exceeds $4a$. [5]

11 A curve has parametric equations

$$x = \cos^2 t, \quad y = \sin^3 t, \quad \text{for } 0 \leq t \leq \frac{1}{2}\pi.$$

(i) Sketch the curve. [2]

(ii) The tangent to the curve at the point $(\cos^2 \theta, \sin^3 \theta)$, where $0 < \theta < \frac{1}{2}\pi$, meets the x - and y -axes at Q and R respectively. The origin is denoted by O . Show that the area of triangle OQR is

$$\frac{1}{12} \sin \theta (3 \cos^2 \theta + 2 \sin^2 \theta)^2. \quad [6]$$

(iii) Show that the area under the curve for $0 \leq t \leq \frac{1}{2}\pi$ is $2 \int_0^{\frac{1}{2}\pi} \cos t \sin^4 t \, dt$, and use the substitution $\sin t = u$ to find this area. [5]



MATHEMATICS

9740/02

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Section A: Pure Mathematics [40 marks]

- 1** Four friends buy three different kinds of fruit in the market. When they get home they cannot remember the individual prices per kilogram, but three of them can remember the total amount that they each paid. The weights of fruit and the total amounts paid are shown in the following table.

	Suresh	Fandi	Cindy	Lee Lian
Pineapples (kg)	1.15	1.20	2.15	1.30
Mangoes (kg)	0.60	0.45	0.90	0.25
Lychees (kg)	0.55	0.30	0.65	0.50
Total amount paid in \$	8.28	6.84	13.05	

Assuming that, for each variety of fruit, the price per kilogram paid by each of the friends is the same, calculate the total amount that Lee Lian paid. [6]

- 2** A sequence u_1, u_2, u_3, \dots is such that $u_1 = 1$ and

$$u_{n+1} = u_n - \frac{2n+1}{n^2(n+1)^2}, \quad \text{for all } n \geq 1.$$

- ⊙ (i) Use the method of mathematical induction to prove that $u_n = \frac{1}{n^2}$. [4]

- (ii) Hence find $\sum_{n=1}^N \frac{2n+1}{n^2(n+1)^2}$. [2]

- (iii) Give a reason why the series in part (ii) is convergent and state the sum to infinity. [2]

- (iv) Use your answer to part (ii) to find $\sum_{n=2}^N \frac{2n-1}{n^2(n-1)^2}$. [2]

- 3** (i) By successively differentiating $(1+x)^n$, find Maclaurin's series for $(1+x)^n$, up to and including the term in x^3 . [4]

- (ii) Obtain the expansion of $(4-x)^{\frac{3}{2}}(1+2x^2)^{\frac{3}{2}}$ up to and including the term in x^3 . [5]

- (iii) Find the set of values of x for which the expansion in part (ii) is valid. [2]

- 4 (i) Find the exact value of $\int_0^{\frac{5}{3}\pi} \sin^2 x \, dx$. Hence find the exact value of $\int_0^{\frac{5}{3}\pi} \cos^2 x \, dx$. [6]
- (ii) The region R is bounded by the curve $y = x^2 \sin x$, the line $x = \frac{1}{2}\pi$ and the part of the x -axis between 0 and $\frac{1}{2}\pi$. Find
- (a) the exact area of R , [5]
- (b) the numerical value of the volume of revolution formed when R is rotated completely about the x -axis, giving your answer correct to 3 decimal places. [2]

Section B: Statistics [60 marks]

- ⑤ (i) Give a real-life example of a situation in which quota sampling could be used. Explain why quota sampling would be appropriate in this situation, and describe briefly any disadvantage that quota sampling has. [4]
- (ii) Explain briefly whether it would be possible to use stratified sampling in the situation you have described in part (i). [1]
- ⑥ In a large population, 24% have a particular gene, A , and 0.3% have gene B . Find the probability that, in a random sample of 10 people from the population, at most 4 have gene A . [2]
- A random sample of 1000 people is taken from the population. Using appropriate approximations, find
- (i) the probability that between 230 and 260 inclusive have gene A , [3]
- (ii) the probability that at least 2 but fewer than 5 have gene B . [2]
- 7 A large number of students in a college have completed a geography project. The time, x hours, taken by a student to complete the project is noted for a random sample of 150 students. The results are summarised by

$$\Sigma x = 4626, \quad \Sigma x^2 = 147\,691.$$

Find unbiased estimates of the population mean and variance. [2]

Test, at the 5% significance level, whether the population mean time for a student to complete the project exceeds 30 hours. [4]

State, giving a reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

- 8 Chickens and turkeys are sold by weight. The masses, in kg, of chickens and turkeys are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Chickens	2.2	0.5
Turkeys	10.5	2.1

Chickens are sold at \$3 per kg and turkeys at \$5 per kg.

- (i) Find the probability that a randomly chosen chicken has a selling price exceeding \$7. [2]
- (ii) Find the probability of the event that both a randomly chosen chicken has a selling price exceeding \$7 and a randomly chosen turkey has a selling price exceeding \$55. [3]
- (iii) Find the probability that the total selling price of a randomly chosen chicken and a randomly chosen turkey is more than \$62. [4]
- (iv) Explain why the answer to part (iii) is greater than the answer to part (ii). [1]

- 9 A group of 12 people consists of 6 married couples.

- (i) The group stand in a line.
 - (a) Find the number of different possible orders. [1]
 - (b) Find the number of different possible orders in which each man stands next to his wife. [3]
- (ii) The group stand in a circle.
 - (a) Find the number of different possible arrangements. [1]
 - (b) Find the number of different possible arrangements if men and women alternate. [2]
 - (c) Find the number of different possible arrangements if each man stands next to his wife and men and women alternate. [2]

- 10 A player throws three darts at a target. The probability that he is successful in hitting the target with his first throw is $\frac{1}{8}$. For each of his second and third throws, the probability of success is

- twice the probability of success on the preceding throw if that throw was successful,
- the same as the probability of success on the preceding throw if that throw was unsuccessful.

Construct a probability tree showing this information. [3]

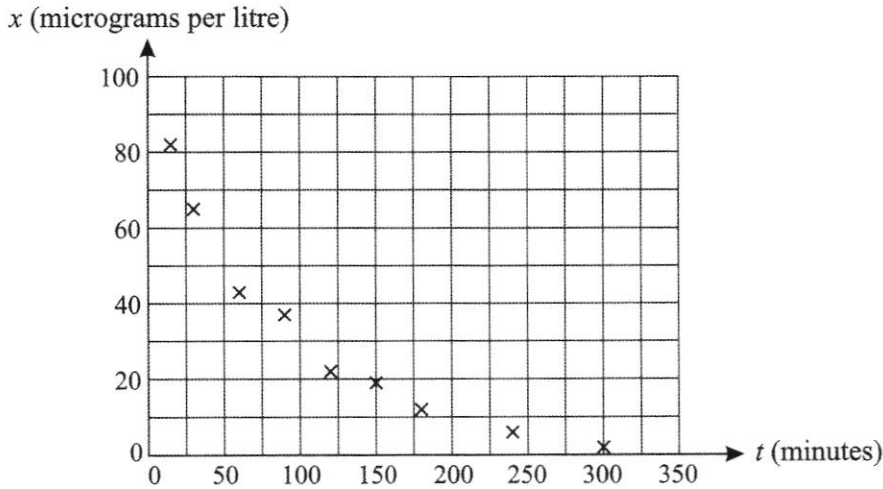
Find

- (i) the probability that all three throws are successful, [2]
- (ii) the probability that at least two throws are successful, [2]
- (iii) the probability that the third throw is successful given that exactly two of the three throws are successful. [4]

- 11 Research is being carried out into how the concentration of a drug in the bloodstream varies with time, measured from when the drug is given. Observations at successive times give the data shown in the following table.

Time (t minutes)	15	30	60	90	120	150	180	240	300
Concentration (x micrograms per litre)	82	65	43	37	22	19	12	6	2

It is given that the value of the product moment correlation coefficient for this data is -0.912 , correct to 3 decimal places. The scatter diagram for the data is shown below.



Calculate the equation of the regression line of x on t .

[2]

Calculate the corresponding estimated value of x when $t = 300$, and comment on the suitability of the linear model.

[2]

The variable y is defined by $y = \ln x$. For the variables y and t ,

(i) calculate the product moment correlation coefficient and comment on its value,

[2]

(ii) calculate the equation of the appropriate regression line.

[3]

Use a regression line to give the best estimate that you can of the time when the drug concentration is 15 micrograms per litre.

[2]

