MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2008

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

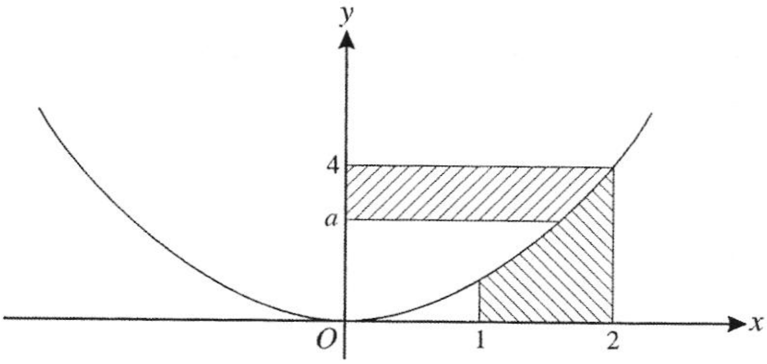
READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1



The diagram shows the curve with equation $y = x^2$. The area of the region bounded by the curve, the lines $x = 1$, $x = 2$ and the x-axis is equal to the area of the region bounded by the curve, the lines $y = a$, $y = 4$ and the y-axis, where $a < 4$. Find the value of a. [4]

2 The nth term of a sequence is given by

$$u_n = n(2n + 1),$$

for $n \geqslant 1$. The sum of the first n terms is denoted by S_n. Use the method of mathematical induction to show that

$$S_n = \tfrac{1}{6}n(n + 1)(4n + 5)$$

for all positive integers n. [5]

3 Points O, A, B are such that $\overrightarrow{OA} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} - \mathbf{j}$, and the point P is such that $OAPB$ is a parallelogram.

(i) Find $\overrightarrow{OP}$. [1]

(ii) Find the size of angle AOB. [3]

(iii) Find the exact area of the parallelogram $OAPB$. [2]

4 **(i)** Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{3x}{x^2 + 1}.$$ [2]

(ii) Find the particular solution of the differential equation for which $y = 2$ when $x = 0$. [1]

(iii) What can you say about the gradient of every solution curve as $x \to \pm\infty$? [1]

(iv) Sketch, on a single diagram, the graph of the solution found in part **(ii)**, together with 2 other members of the family of solution curves. [3]

5 **(i)** Find the exact value of $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{1+9x^2}\,dx$. [3]

(ii) Find, in terms of n and e, $\displaystyle\int_1^{e} x^n \ln x\,dx$, where $n \neq -1$. [4]

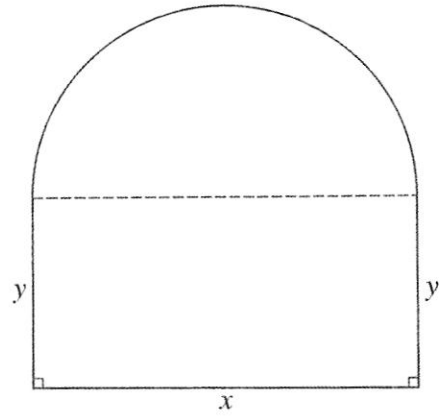
6 **(a)** In the triangle ABC, $AB = 1$, $BC = 3$ and angle $ABC = \theta$ radians. Given that θ is a sufficiently small angle, show that

$$AC \approx (4 + 3\theta^2)^{\frac{1}{2}} \approx a + b\theta^2,$$

for constants a and b to be determined. [5]

(b) Given that $f(x) = \tan(2x + \frac{1}{4}\pi)$, find $f(0)$, $f'(0)$ and $f''(0)$. Hence find the first 3 terms in the Maclaurin series of $f(x)$. [5]

7



A new flower-bed is being designed for a large garden. The flower-bed will occupy a rectangle x m by y m together with a semicircle of diameter x m, as shown in the diagram. A low wall will be built around the flower-bed. The time needed to build the wall will be 3 hours per metre for the straight parts and 9 hours per metre for the semicircular part. Given that a total time of 180 hours is taken to build the wall, find, using differentiation, the values of x and y which give a flower-bed of maximum area. [10]

8 A graphic calculator is **not** to be used in answering this question.

(i) It is given that $z_1 = 1 + (\sqrt{3})i$. Find the value of z_1^3, showing clearly how you obtain your answer. [3]

(ii) Given that $1 + (\sqrt{3})i$ is a root of the equation

$$2z^3 + az^2 + bz + 4 = 0,$$

find the values of the real numbers a and b. [4]

(iii) For these values of a and b, solve the equation in part **(ii)**, and show all the roots on an Argand diagram. [4]

9 It is given that

$$f(x) = \frac{ax + b}{cx + d},$$

for non-zero constants a, b, c and d.

(i) Given that $ad - bc \neq 0$, show by differentiation that the graph of $y = f(x)$ has no turning points. [3]

(ii) What can be said about the graph of $y = f(x)$ when $ad - bc = 0$? [2]

(iii) Deduce from part **(i)** that the graph of

$$y = \frac{3x - 7}{2x + 1}$$

has a positive gradient at all points of the graph. [1]

(iv) On separate diagrams, draw sketches of the graphs of

(a) $y = \dfrac{3x - 7}{2x + 1}$,

⑤ **(b)** $y^2 = \dfrac{3x - 7}{2x + 1}$,

including the coordinates of the points where the graphs cross the axes and the equations of any asymptotes. [5]

10 **(i)** A student saves \$10 on 1 January 2009. On the first day of each subsequent month she saves \$3 more than in the previous month, so that she saves \$13 on 1 February 2009, \$16 on 1 March 2009, and so on. On what date will she first have saved over \$2000 in total? [5]

(ii) A second student puts \$10 on 1 January 2009 into a bank account which pays compound interest at a rate of 2% per month on the last day of each month. She puts a further \$10 into the account on the first day of each subsequent month.

(a) How much compound interest has her original \$10 earned at the end of 2 years? [2]

(b) How much in total is in the account at the end of 2 years? [3]

(c) After how many complete months will the total in the account first exceed \$2000? [4]

11 The equations of three planes p_1, p_2, p_3 are

$$\begin{aligned} 2x - 5y + 3z &= 3, \\ 3x + 2y - 5z &= -5, \\ 5x + \lambda y + 17z &= \mu, \end{aligned}$$

respectively, where λ and μ are constants. When $\lambda = -20.9$ and $\mu = 16.6$, find the coordinates of the point at which these planes meet. [2]

The planes p_1 and p_2 intersect in a line l.

(i) Find a vector equation of l. [4]

(ii) Given that all three planes meet in the line l, find λ and μ. [3]

(iii) Given instead that the three planes have no point in common, what can be said about the values of λ and μ? [2]

(iv) Find the cartesian equation of the plane which contains l and the point $(1, -1, 3)$. [4]