

MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

MATHEMATICS

Paper 1

9740/01 October/November 2008

3 hours

Additional Materials: Answer Paper Graph paper List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.



 \odot 2 The *n*th term of a sequence is given by

 $u_n = n(2n+1),$

for $n \ge 1$. The sum of the first *n* terms is denoted by S_n . Use the method of mathematical induction to show that

$$S_n = \frac{1}{6}n(n+1)(4n+5)$$

for all positive integers n.

(ii) Find the size of angle AOB.

- 3 Points O, A, B are such that $\overrightarrow{OA} = \mathbf{i} + 4\mathbf{j} 3\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} \mathbf{j}$, and the point P is such that OAPB is a parallelogram.
 - (i) Find \overrightarrow{OP} . [1]
 - (iii) Find the exact area of the parallelogram *OAPB*.
- 4 (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x}{x^2 + 1}.$$
[2]

- (ii) Find the particular solution of the differential equation for which y = 2 when x = 0. [1]
- (iii) What can you say about the gradient of every solution curve as $x \to \pm \infty$? [1]
- (iv) Sketch, on a single diagram, the graph of the solution found in part (ii), together with 2 other members of the family of solution curves.

[5]

[1]

[2]

5

7

(i)

Find the exact value of
$$\int_{0}^{\frac{1}{\sqrt{3}}} \frac{1}{1+9x^2} dx.$$

(ii) Find, in terms of *n* and e, $\int_{1}^{e} x^{n} \ln x \, dx$, where $n \neq -1$.

6 (a) In the triangle ABC, AB = 1, BC = 3 and angle $ABC = \theta$ radians. Given that θ is a sufficiently small angle, show that

$$AC \approx \left(4+3\theta^2\right)^{\frac{1}{2}} \approx a+b\theta^2,$$

for constants a and b to be determined.

(b) Given that $f(x) = \tan(2x + \frac{1}{4}\pi)$, find f(0), f'(0) and f''(0). Hence find the first 3 terms in the Maclaurin series of f(x). [5]

A new flower-bed is being designed for a large garden. The flower-bed will occupy a rectangle x m by y m together with a semicircle of diameter x m, as shown in the diagram. A low wall will be built around the flower-bed. The time needed to build the wall will be 3 hours per metre for the straight parts and 9 hours per metre for the semicircular part. Given that a total time of 180 hours is taken to build the wall, find, using differentiation, the values of x and y which give a flower-bed of maximum area. [10]

- 8 A graphic calculator is **not** to be used in answering this question.
 - (i) It is given that $z_1 = 1 + (\sqrt{3})i$. Find the value of z_1^3 , showing clearly how you obtain your answer. [3]
 - (ii) Given that $1 + (\sqrt{3})i$ is a root of the equation

$$2z^3 + az^2 + bz + 4 = 0,$$

find the values of the real numbers a and b.

(iii) For these values of a and b, solve the equation in part (ii), and show all the roots on an Argand diagram.



[3]

[4]

[5]

[4]

9 It is given that

$$f(x) = \frac{ax+b}{cx+d},$$

for non-zero constants a, b, c and d.

- (i) Given that $ad bc \neq 0$, show by differentiation that the graph of y = f(x) has no turning points.
- (ii) What can be said about the graph of y = f(x) when ad bc = 0?
- (iii) Deduce from part (i) that the graph of

$$y = \frac{3x - 7}{2x + 1}$$

has a positive gradient at all points of the graph.

(iv) On separate diagrams, draw sketches of the graphs of

(a)
$$y = \frac{3x-7}{2x+1}$$
,

(b)
$$y^2 = \frac{3x-7}{2x+1}$$
,

including the coordinates of the points where the graphs cross the axes and the equations of any asymptotes. [5]

- (i) A student saves \$10 on 1 January 2009. On the first day of each subsequent month she saves \$3 more than in the previous month, so that she saves \$13 on 1 February 2009, \$16 on 1 March 2009, and so on. On what date will she first have saved over \$2000 in total? [5]
 - (ii) A second student puts \$10 on 1 January 2009 into a bank account which pays compound interest at a rate of 2% per month on the last day of each month. She puts a further \$10 into the account on the first day of each subsequent month.
 - (a) How much compound interest has her original \$10 earned at the end of 2 years? [2]
 - (b) How much in total is in the account at the end of 2 years?
 - (c) After how many complete months will the total in the account first exceed \$2000?

[1]

[3]

[4]

[3]

[2]

11 The equations of three planes p_1, p_2, p_3 are

$$2x - 5y + 3z = 3,3x + 2y - 5z = -5,5x + \lambda y + 17z = \mu,$$

respectively, where λ and μ are constants. When $\lambda = -20.9$ and $\mu = 16.6$, find the coordinates of the point at which these planes meet. [2]

The planes p_1 and p_2 intersect in a line l.

 (\mathfrak{G}) (i) Find a vector equation of l.

A Starte

[4]

[3]

- \mathfrak{S} (ii) Given that all three planes meet in the line *l*, find λ and μ .
- (iii) Given instead that the three planes have no point in common, what can be said about the values of λ and μ ? [2]
 - (iv) Find the cartesian equation of the plane which contains l and the point (1, -1, 3). [4]

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MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

MATHEMATICS

Paper 2

October/November 2008 3 hours

9740/02

Additional Materials: Answer Paper Graph paper List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

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Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

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At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question. 1 Let $f(x) = e^x \sin x$.

2

- (i) Sketch the graph of y = f(x) for $-3 \le x \le 3$.
- (ii) Find the series expansion of f(x) in ascending powers of x, up to and including the term in x^3 .

Denote the answer to part (ii) by g(x).

- (iii) On the same diagram as in part (i), sketch the graph of y = g(x). Label the two graphs clearly.
- (iv) Find, for $-3 \le x \le 3$, the set of values of x for which the value of g(x) is within ±0.5 of the value of f(x). [3]



The diagram shows the curve C with equation $y^2 = x\sqrt{(1-x)}$. The region enclosed by C is denoted by R.

- (i) Write down an integral that gives the area of R, and evaluate this integral numerically. [3]
- (ii) The part of R above the x-axis is rotated through 2π radians about the x-axis. By using the substitution u = 1 x, or otherwise, find the exact value of the volume obtained. [3]
- (iii) Find the exact x-coordinate of the maximum point of C.

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[2]

[3]

[1]

[3]

3 (a) The complex number w has modulus r and argument θ , where $0 < \theta < \frac{1}{2}\pi$, and w^* denotes the conjugate of w. State the modulus and argument of p, where $p = \frac{w}{w^*}$. [2]

Given that p^5 is real and positive, find the possible values of θ . [2]

- (b) The complex number z satisfies the relations $|z| \le 6$ and |z| = |z 8 6i|.
 - (i) Illustrate both of these relations on a single Argand diagram. [3]
 - (ii) Find the greatest and least possible values of arg z, giving your answers in radians correct to 3 decimal places.
- 4 The function f is defined by $f: x \mapsto (x-4)^2 + 1$ for $x \in \mathbb{R}$, x > 4.
 - (i) Sketch the graph of y = f(x). Your sketch should indicate the position of the graph in relation to the origin. [2]
 - (ii) Find $f^{-1}(x)$, stating the domain of f^{-1} .
 - (iii) On the same diagram as in part (i), sketch the graph of $y = f^{-1}(x)$. [1]
 - (iv) Write down the equation of the line in which the graph of y = f(x) must be reflected in order to obtain the graph of $y = f^{-1}(x)$, and hence find the exact solution of the equation $f(x) = f^{-1}(x)$. [5]

Section B: Statistics [60 marks]

S A school has 950 pupils. A sample of 50 pupils is to be chosen to take part in a survey. Describe how the sample could be chosen using systematic sampling.
 [2]

The purpose of the survey is to investigate pupils' opinions about the sports facilities available at the school. Give a reason why a stratified sample might be preferable in this context. [2]

 \odot 6 In mineral water from a certain source, the mass of calcium, X mg, in a one-litre bottle is a normally distributed random variable with mean μ . Based on observations over a long period, it is known that $\mu = 78$. Following a period of extreme weather, 15 randomly chosen bottles of the water were analysed. The masses of calcium in the bottles are summarised by

$$\Sigma x = 1026.0, \quad \Sigma x^2 = 77265.90.$$

Test, at the 5% significance level, whether the mean mass of calcium in a bottle has changed. [6]

[3]

7 A computer game simulates a tennis match between two players, *A* and *B*. The match consists of at most three sets. Each set is won by either *A* or *B*, and the match is won by the first player to win two sets.

The simulation uses the following rules.

- The probability that A wins the first set is 0.6.
- For each set after the first, the conditional probability that A wins that set, given that A won the preceding set, is 0.7.
- For each set after the first, the conditional probability that *B* wins that set, given that *B* won the preceding set, is 0.8.

Calculate the probability that

(i)	A wins the second set,	[2]
(ii)	A wins the match,	[3]
(iii)	B won the first set, given that A wins the match.	[3]

8 A certain metal discolours when exposed to air. To protect the metal against discolouring, it is treated with a chemical. In an experiment, different quantities, x ml, of the chemical were applied to standard samples of the metal, and the times, t hours, for the metal to discolour were measured. The results are given in the table.

x	1.2	2.0	2.7	3.8	4.8	5.6	6.9
t	2.2	4.5	5.8	7.3	7.6	9.0	9.9

- (i) Calculate the product moment correlation coefficient between *x* and *t*, and explain whether your answer suggests that a linear model is appropriate. [3]
- (ii) Draw a scatter diagram for the data.

One of the values of t appears to be incorrect.

- (iii) Indicate the corresponding point on your diagram by labelling it *P*, and explain why the scatter diagram for the remaining points may be consistent with a model of the form $t = a + b \ln x$. [2]
- (iv) Omitting *P*, calculate least squares estimates of *a* and *b* for the model $t = a + b \ln x$. [2]
- (v) Assume that the value of x at P is correct. Estimate the value of t for this value of x. [1]
- (vi) Comment on the use of the model in part (iv) in predicting the value of t when x = 8.0. [1]

1

[1]

A shop sells two types of piano, 'grand' and 'upright'. The mean number of grand pianos sold in a week is 1.8. Use a Poisson distribution to find the probability that in a given week at least 4 grand pianos are sold.

The mean number of upright pianos sold in a week is 2.6. The sales of the two types of piano are independent. Use a Poisson distribution to find the probability that in a given week the total number of pianos sold is exactly 4. [2]

Use a normal approximation to the Poisson distribution to find the probability that the number of grand pianos sold in a year of 50 weeks is less than 80. [4]

Explain why the Poisson distribution may not be a good model for the number of grand pianos sold in a year. [2]

10 A group of diplomats is to be chosen to represent three islands, K, L and M. The group is to consist of 8 diplomats and is chosen from a set of 12 diplomats consisting of 3 from K, 4 from L and 5 from M. Find the number of ways in which the group can be chosen if it includes

(i)	2 diplomats from K , 3 from L and 3 from M ,	[2]
(ii)	diplomats from L and M only,	[2]
(iii)	at least 4 diplomats from M ,	[2]
(iv)	at least 1 diplomat from each island.	[4]

- 11 The random variable X has the distribution N(50, 8^2). Given that X_1 and X_2 are two independent observations of X, find
 - (i) $P(X_1 + X_2 > 120),$ [2]

(ii)
$$P(X_1 > X_2 + 15)$$
. [3]

The random variable *Y* is related to *X* by the formula Y = aX + b, where *a* and *b* are constants with a > 0. Given that P(Y < 74) = P(Y > 146) = 0.0668, find the values of E(Y) and Var(Y), and hence find the values of *a* and *b*. [7]

