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MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

# MATHEMATICS

Paper 1

9740/01 October/November 2009

3 hours

Additional Materials: Answer Paper Graph paper List of Formulae (MF15)

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.

- 1 (i) The first three terms of a sequence are given by  $u_1 = 10$ ,  $u_2 = 6$ ,  $u_3 = 5$ . Given that  $u_n$  is a quadratic polynomial in *n*, find  $u_n$  in terms of *n*. [4]
  - (ii) Find the set of values of *n* for which  $u_n$  is greater than 100.
- 2 Find the exact value of p such that

$$\int_{0}^{1} \frac{1}{4 - x^{2}} \, \mathrm{d}x = \int_{0}^{\frac{1}{2p}} \frac{1}{\sqrt{1 - p^{2}x^{2}}} \, \mathrm{d}x.$$
 [5]

[2]

[3]

- 3 (i) Show that  $\frac{1}{n-1} \frac{2}{n} + \frac{1}{n+1} = \frac{A}{n^3 n}$ , where A is a constant to be found. [2]
  - (ii) Hence find  $\sum_{r=2}^{n} \frac{1}{r^3 r}$ . (There is no need to express your answer as a single algebraic fraction.) [3]

(iii) Give a reason why the series 
$$\sum_{r=2}^{\infty} \frac{1}{r^3 - r}$$
 converges, and write down its value. [2]

4 It is given that

$$f(x) = \begin{cases} 7 - x^2 & \text{for } 0 < x \le 2, \\ 2x - 1 & \text{for } 2 < x \le 4, \end{cases}$$

and that f(x) = f(x + 4) for all real values of x.

- (i) Evaluate f(27) + f(45). [2]
- (ii) Sketch the graph of y = f(x) for  $-7 \le x \le 10$ .

(iii) Find 
$$\int_{-4}^{3} f(x) dx$$
. [3]

© 5 Use the method of mathematical induction to prove that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$
[4]

Find 
$$\sum_{r=n+1}^{2n} r^2$$
, giving the answer in fully factorised form. [4]

- The curve  $C_1$  has equation  $y = \frac{x-2}{x+2}$ . The curve  $C_2$  has equation  $\frac{x^2}{6} + \frac{y^2}{3} = 1$ . 6
  - (i) Sketch  $C_1$  and  $C_2$  on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]
  - (ii) Show algebraically that the x-coordinates of the points of intersection of  $C_1$  and  $C_2$  satisfy the equation  $2(x-2)^2 = (x+2)^2(6-x^2)$ . [2]
  - (iii) Use your calculator to find these x-coordinates.
  - (i) Given that  $f(x) = e^{\cos x}$ , find f(0), f'(0) and f''(0). Hence write down the first two non-zero terms in the Maclaurin series for f(x). Give the coefficients in terms of e. [5]
    - (ii) Given that the first two non-zero terms in the Maclaurin series for f(x) are equal to the first two non-zero terms in the series expansion of  $\frac{1}{a+bx^2}$ , where a and b are constants, find a and b in terms of e. [4]
- 8 Two musical instruments, A and B, consist of metal bars of decreasing lengths.
  - (i) The first bar of instrument A has length 20 cm and the lengths of the bars form a geometric progression. The 25th bar has length 5 cm. Show that the total length of all the bars must be less than 357 cm, no matter how many bars there are. [4]

Instrument B consists of only 25 bars which are identical to the first 25 bars of instrument A.

- (ii) Find the total length,  $L \, \text{cm}$ , of all the bars of instrument B and the length of the 13th bar. [3]
- (iii) Unfortunately the manufacturer misunderstands the instructions and constructs instrument B wrongly, so that the lengths of the bars are in arithmetic progression with common difference d cm. If the total length of the 25 bars is still L cm and the length of the 25th bar is still 5 cm, find the value of d and the length of the longest bar. [4]
- (i) Solve the equation

$$z^7 - (1 + i) = 0$$

giving the roots in the form  $re^{i\alpha}$ , where r > 0 and  $-\pi < \alpha \le \pi$ . [5]

- (ii) Show the roots on an Argand diagram.
- (iii) The roots represented by  $z_1$  and  $z_2$  are such that  $0 < \arg(z_1) < \arg(z_2) < \frac{1}{2}\pi$ . Explain why the locus of all points z such that  $|z - z_1| = |z - z_2|$  passes through the origin. Draw this locus on your Argand diagram and find its exact cartesian equation. [5]

[2]

[2]

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- 10 The planes  $p_1$  and  $p_2$  have equations  $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 1$  and  $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 2$  respectively, and meet in a line *l*.
  - (i) Find the acute angle between  $p_1$  and  $p_2$ .
  - (ii) Find a vector equation of l.
  - (iii) The plane  $p_3$  has equation 2x + y + 3z 1 + k(-x + 2y + z 2) = 0. Explain why *l* lies in  $p_3$  for any constant *k*. Hence, or otherwise, find a cartesian equation of the plane in which both *l* and the point (2, 3, 4) lie. [5]
- 11 The curve C has equation y = f(x), where  $f(x) = xe^{-x^2}$ 
  - (i) Sketch the curve C.
    - (ii) Find the exact coordinates of the turning points on the curve.
  - (iii) Use the substitution  $u = x^2$  to find  $\int_0^n f(x) dx$ , for n > 0. Hence find the area of the region between the curve and the positive x-axis. [4]
  - (iv) Find the exact value of  $\int_{-2}^{2} |f(x)| dx$ . [2]
  - (v) Find the volume of revolution when the region bounded by the curve, the lines x = 0, x = 1 and the x-axis is rotated completely about the x-axis. Give your answer correct to 3 significant figures.

[2]

[3]

[4]

[2]

[4]



MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

# MATHEMATICS

Paper 2

9740/02

October/November 2009 3 hours

Additional Materials: Answer P Graph pa

Answer Paper Graph paper List of Formulae (MF15)

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

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At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.

#### Section A: Pure Mathematics [40 marks]

1 The curve C has parametric equations

 $x = t^2 + 4t$ ,  $y = t^3 + t^2$ .

(i) Sketch the curve for  $-2 \le t \le 1$ . [1]

The tangent to the curve at the point P where t = 2 is denoted by l.

- (ii) Find the cartesian equation of l.
- (iii) The tangent l meets C again at the point Q. Use a non-calculator method to find the coordinates of Q. [4]

[3]

[2]

- 2 Relative to the origin O, two points A and B have position vectors given by  $\mathbf{a} = 14\mathbf{i} + 14\mathbf{j} + 14\mathbf{k}$  and  $\mathbf{b} = 11\mathbf{i} 13\mathbf{j} + 2\mathbf{k}$  respectively.
  - (i) The point P divides the line AB in the ratio 2 : 1. Find the coordinates of P. [2]
  - (ii) Show that AB and OP are perpendicular.
  - (iii) The vector **c** is a unit vector in the direction of  $\overrightarrow{OP}$ . Write **c** as a column vector, and give the geometrical meaning of  $|\mathbf{a.c}|$ . [2]
  - (iv) Find  $\mathbf{a} \times \mathbf{p}$ , where  $\mathbf{p}$  is the vector  $\overrightarrow{OP}$ , and give the geometrical meaning of  $|\mathbf{a} \times \mathbf{p}|$ . Hence write down the area of triangle *OAP*. [4]
- 3 The function f is defined by

$$f: x \mapsto \frac{ax}{bx-a}, \quad \text{for } x \in \mathbb{R}, \ x \neq \frac{a}{b},$$

where a and b are non-zero constants.

- (i) Find  $f^{-1}(x)$ . Hence or otherwise find  $f^{2}(x)$  and state the range of  $f^{2}$ . [5]
- (ii) The function g is defined by  $g: x \mapsto \frac{1}{x}$  for all real non-zero x. State whether the composite function fg exists, justifying your answer. [2]
- (iii) Solve the equation  $f^{-1}(x) = x$ . [3]
- 4 Two scientists are investigating the change of a certain population of size *n* thousand at time *t* years.
  - (i) One scientist suggests that *n* and *t* are related by the differential equation  $\frac{d^2n}{dt^2} = 10 6t$ . Find the general solution of this differential equation. Sketch three members of the family of solution curves, given that n = 100 when t = 0. [5]
  - (ii) The other scientist suggests that n and t are related by the differential equation  $\frac{dn}{dt} = 3 0.02n$ . Find n in terms of t, given again that n = 100 when t = 0. Explain in simple terms what will eventually happen to the population using this model. [7]

### Section B: Statistics [60 marks]

- **G** 5 A cinema manager wishes to take a survey of opinions of cinema-goers. Describe how a quota sample of size 100 might be obtained, and state one disadvantage of quota sampling. [3]
  - 6 The table gives the world record time, in seconds above 3 minutes 30 seconds, for running 1 mile as at 1st January in various years.

Year, x	1930	1940	1950	1960	1970	1980	1990	2000
Time, t	40.4	36.4	31.3	24.5	21.1	19.0	16.3	13.1

- (i) Draw a scatter diagram to illustrate the data.
- (ii) Comment on whether a linear model would be appropriate, referring both to the scatter diagram and the context of the question. [2]
- (iii) Explain why in this context a quadratic model would probably not be appropriate for long-term predictions. [1]
- (iv) Fit a model of the form  $\ln t = a + bx$  to the data, and use it to predict the world record time as at 1st January 2010. Comment on the reliability of your prediction. [3]
- 7 A company buys p% of its electronic components from supplier A and the remaining (100 - p)% from supplier B. The probability that a randomly chosen component supplied by A is faulty is 0.05. The probability that a randomly chosen component supplied by B is faulty is 0.03.
  - [2] (i) Given that p = 25, find the probability that a randomly chosen component is faulty.
  - (ii) For a general value of p, the probability that a randomly chosen component that is faulty was supplied by A is denoted by f(p). Show that  $f(p) = \frac{0.05p}{0.02p+3}$ . Prove by differentiation that f is an increasing function for  $0 \le p \le 100$ , and explain what this statement means in the context of the question. [6]

#### Find the number of ways in which the letters of the word ELEVATED can be arranged if

(i)	there are no restrictions,	[1]
(ii)	T and D must not be next to one another,	[2]
(iii)	consonants (L, V, T, D) and vowels (E, A) must alternate,	[3]
(iv)	between any two Es there must be at least 2 other letters.	[3]

[2]

8

- 9 The thickness in cm of a mechanics textbook is a random variable with the distribution  $N(2.5, 0.1^2)$ .
  - (i) The mean thickness of *n* randomly chosen mechanics textbooks is denoted by  $\overline{M}$  cm. Given that  $P(\overline{M} > 2.53) = 0.0668$ , find the value of *n*. [3]

The thickness in cm of a statistics textbook is a random variable with the distribution  $N(2.0, 0.08^2)$ .

- (ii) Calculate the probability that 21 mechanics textbooks and 24 statistics textbooks will fit onto a bookshelf of length 1 m. State clearly the mean and variance of any normal distribution you use in your calculation.
- (iii) Calculate the probability that the total thickness of 4 statistics textbooks is less than three times the thickness of 1 mechanics textbook. State clearly the mean and variance of any normal distribution you use in your calculation.
- (iv) State an assumption needed for your calculations in parts (ii) and (iii). [1]
- 3 10 A company supplies sugar in small packets. The mass of sugar in one packet is denoted by X grams. The masses of a random sample of 9 packets are summarised by

$$\Sigma x = 86.4, \quad \Sigma x^2 = 835.92.$$

[2]

[2]

[2]

(i) Calculate unbiased estimates of the mean and variance of X.

The mean mass of sugar in a packet is claimed to be 10 grams. The company directors want to know whether the sample indicates that this claim is incorrect.

- (ii) Stating a necessary assumption, carry out a *t*-test at the 5% significance level. Explain why the Central Limit Theorem does not apply in this context.
- (iii) Suppose now that the population variance of X is known, and that the assumption made in part (ii) is still valid. What change would there be in carrying out the test? [1]
- 11 A fixed number, n, of cars is observed and the number of those cars that are red is denoted by R.
  - (i) State, in context, two assumptions needed for R to be well modelled by a binomial distribution.

Assume now that *R* has the distribution B(n, p).

- (ii) Given that n = 20 and p = 0.15, find  $P(4 \le R < 8)$ .
- (iii) Given that n = 240 and p = 0.3, find P(R < 60) using a suitable approximation, which should be clearly stated. [3]
- (iv) Given that n = 240 and p = 0.02, find P(R = 3) using a suitable approximation, giving your answer correct to 4 decimal places and explaining why the approximation is appropriate in this case. [3]
  - (v) Given that n = 20 and P(R = 0 or 1) = 0.2, write down an equation for the value of p, and find this value numerically. [2]