MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2009

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 **(i)** The first three terms of a sequence are given by $u_1 = 10$, $u_2 = 6$, $u_3 = 5$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [4]

(ii) Find the set of values of n for which u_n is greater than 100. [2]

2 Find the exact value of p such that

$$\int_0^1 \frac{1}{4-x^2}\,dx = \int_0^{\frac{1}{2p}} \frac{1}{\sqrt{(1-p^2x^2)}}\,dx. \qquad [5]$$

3 **(i)** Show that $\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} = \frac{A}{n^3-n}$, where A is a constant to be found. [2]

(ii) Hence find $\sum_{r=2}^{n} \frac{1}{r^3-r}$. (There is no need to express your answer as a single algebraic fraction.) [3]

(iii) Give a reason why the series $\sum_{r=2}^{\infty} \frac{1}{r^3-r}$ converges, and write down its value. [2]

4 It is given that

$$f(x) = \begin{cases} 7-x^2 & \text{for } 0 < x \leqslant 2, \\ 2x-1 & \text{for } 2 < x \leqslant 4, \end{cases}$$

and that $f(x) = f(x+4)$ for all real values of x.

(i) Evaluate $f(27) + f(45)$. [2]

(ii) Sketch the graph of $y = f(x)$ for $-7 \leqslant x \leqslant 10$. [3]

(iii) Find $\int_{-4}^{3} f(x)\,dx$. [3]

5 Use the method of mathematical induction to prove that

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1). \qquad [4]$$

Find $\sum_{r=n+1}^{2n} r^2$, giving the answer in fully factorised form. [4]

6 The curve C_1 has equation $y = \dfrac{x-2}{x+2}$. The curve C_2 has equation $\dfrac{x^2}{6} + \dfrac{y^2}{3} = 1$.

(i) Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

(ii) Show algebraically that the x-coordinates of the points of intersection of C_1 and C_2 satisfy the equation $2(x-2)^2 = (x+2)^2(6-x^2)$. [2]

(iii) Use your calculator to find these x-coordinates. [2]

7 **(i)** Given that $f(x) = e^{\cos x}$, find $f(0)$, $f'(0)$ and $f''(0)$. Hence write down the first two non-zero terms in the Maclaurin series for $f(x)$. Give the coefficients in terms of e. [5]

(ii) Given that the first two non-zero terms in the Maclaurin series for $f(x)$ are equal to the first two non-zero terms in the series expansion of $\dfrac{1}{a+bx^2}$, where a and b are constants, find a and b in terms of e. [4]

8 Two musical instruments, A and B, consist of metal bars of decreasing lengths.

(i) The first bar of instrument A has length 20 cm and the lengths of the bars form a geometric progression. The 25th bar has length 5 cm. Show that the total length of all the bars must be less than 357 cm, no matter how many bars there are. [4]

Instrument B consists of only 25 bars which are identical to the first 25 bars of instrument A.

(ii) Find the total length, L cm, of all the bars of instrument B and the length of the 13th bar. [3]

(iii) Unfortunately the manufacturer misunderstands the instructions and constructs instrument B wrongly, so that the lengths of the bars are in arithmetic progression with common difference d cm. If the total length of the 25 bars is still L cm and the length of the 25th bar is still 5 cm, find the value of d and the length of the longest bar. [4]

9 **(i)** Solve the equation

$$z^7 - (1+i) = 0,$$

giving the roots in the form $re^{i\alpha}$, where $r > 0$ and $-\pi < \alpha \leqslant \pi$. [5]

(ii) Show the roots on an Argand diagram. [2]

(iii) The roots represented by z_1 and z_2 are such that $0 < \arg(z_1) < \arg(z_2) < \frac{1}{2}\pi$. Explain why the locus of all points z such that $|z - z_1| = |z - z_2|$ passes through the origin. Draw this locus on your Argand diagram and find its exact cartesian equation. [5]

10 The planes p_1 and p_2 have equations $\mathbf{r}.\begin{pmatrix}2\\1\\3\end{pmatrix} = 1$ and $\mathbf{r}.\begin{pmatrix}-1\\2\\1\end{pmatrix} = 2$ respectively, and meet in a line l.

(i) Find the acute angle between p_1 and p_2. [3]

(ii) Find a vector equation of l. [4]

(iii) The plane p_3 has equation $2x + y + 3z - 1 + k(-x + 2y + z - 2) = 0$. Explain why l lies in p_3 for any constant k. Hence, or otherwise, find a cartesian equation of the plane in which both l and the point (2, 3, 4) lie. [5]

11 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = xe^{-x^2}$

(i) Sketch the curve C. [2]

(ii) Find the exact coordinates of the turning points on the curve. [4]

(iii) Use the substitution $u = x^2$ to find $\int_0^n \mathrm{f}(x)\,\mathrm{d}x$, for $n > 0$. Hence find the area of the region between the curve and the positive x-axis. [4]

(iv) Find the exact value of $\int_{-2}^{2} |\mathrm{f}(x)|\,\mathrm{d}x$. [2]

(v) Find the volume of revolution when the region bounded by the curve, the lines $x = 0$, $x = 1$ and the x-axis is rotated completely about the x-axis. Give your answer correct to 3 significant figures. [2]