

MINISTRY OF EDUCATION, SINGAPORE in collaboration with UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE General Certificate of Education Advanced Level Higher 2

## MATHEMATICS

Paper 1

# 9740/01

October/November 2010

3 hours

Additional Materials: Answer Paper Graph paper List of Formulae (MF15)

### READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question.  $\mathbf{a} = 2p\mathbf{i} + 3p\mathbf{j} + 6p\mathbf{k}$  and  $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ ,

where p > 0. It is given that  $|\mathbf{a}| = |\mathbf{b}|$ .

- (i) Find the exact value of *p*.
- (ii) Show that  $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} \mathbf{b}) = 0$ .
- 2 (i) Find the first three terms of the Maclaurin series for e<sup>x</sup>(1 + sin 2x). [You may use standard results given in the List of Formulae (MF15).] [3]
  - (ii) It is given that the first two terms of this series are equal to the first two terms in the series expansion, in ascending powers of x, of  $(1 + \frac{4}{3}x)^n$ . Find n and show that the third terms in each of these series are equal. [3]

[2]

[3]

[3]

[2]

3 The sum,  $S_n$ , of the first *n* terms of a sequence  $u_1, u_2, u_3, \ldots$  is given by

$$S_n = n(2n+c),$$

where c is a constant.

- (i) Find  $u_n$  in terms of c and n.
- $\odot$  (ii) Find a recurrence relation of the form  $u_{n+1} = f(u_n)$ .
- 4 (i) Given that  $x^2 y^2 + 2xy + 4 = 0$ , find  $\frac{dy}{dx}$  in terms of x and y. [4]
  - (ii) For the curve with equation  $x^2 y^2 + 2xy + 4 = 0$ , find the coordinates of each point at which the tangent is parallel to the x-axis. [4]
- 5 The curve with equation  $y = x^3$  is transformed by a translation of 2 units in the positive x-direction, followed by a stretch with scale factor  $\frac{1}{2}$  parallel to the y-axis, followed by a translation of 6 units in the negative y-direction.
  - (i) Find the equation of the new curve in the form y = f(x) and the exact coordinates of the points where this curve crosses the *x* and *y*-axes. Sketch the new curve. [5]
  - (ii) On the same diagram, sketch the graph of  $y = f^{-1}(x)$ , stating the exact coordinates of the points where the graph crosses the x- and y-axes. [3]



The diagram shows the curve with equation  $y = x^3 - 3x + 1$  and the line with equation y = 1. The curve crosses the x-axis at  $x = \alpha$ ,  $x = \beta$  and  $x = \gamma$  and has turning points at x = -1 and x = 1.

(i) Find the values of  $\beta$  and  $\gamma$ , giving your answers correct to 3 decimal places. [2]

(ii) Find the area of the region bounded by the curve and the x-axis between  $x = \beta$  and  $x = \gamma$ . [2]

(iii) Use a non-calculator method to find the area of the shaded region between the curve and the line. [4]

(iv) Find the set of values of k for which the equation  $x^3 - 3x + 1 = k$  has three real distinct roots. [2]

7 A bottle containing liquid is taken from a refrigerator and placed in a room where the temperature is a constant 20 °C. As the liquid warms up, the rate of increase of its temperature  $\theta$  °C after time *t* minutes is proportional to the temperature difference  $(20 - \theta)$  °C. Initially the temperature of the liquid is 10 °C and the rate of increase of the temperature is 1 °C per minute. By setting up and solving a differential equation, show that  $\theta = 20 - 10e^{-\frac{1}{10}t}$ . [7]

Find the time it takes the liquid to reach a temperature of 15 °C, and state what happens to  $\theta$  for large values of *t*. Sketch a graph of  $\theta$  against *t*. [4]

8 The complex numbers  $z_1$  and  $z_2$  are given by  $1 + i\sqrt{3}$  and -1 - i respectively.

- (i) Express each of  $z_1$  and  $z_2$  in polar form  $r(\cos \theta + i \sin \theta)$ , where r > 0 and  $-\pi < \theta \le \pi$ . Give r and  $\theta$  in exact form. [2]
- (ii) Find the complex conjugate of  $\frac{z_1}{z_2}$  in exact polar form. [3]

(iii) On a single Argand diagram, sketch the loci

- (a)  $|z-z_1|=2$ ,
- (b)  $\arg(z z_2) = \frac{1}{4}\pi$ . [4]

 $\odot$  (iv) Find where the locus  $|z - z_1| = 2$  meets the positive real axis.

6

[2]



A company requires a box made of cardboard of negligible thickness to hold  $300 \text{ cm}^3$  of powder when full. The length of the box is 3x cm, the width is x cm and the height is y cm. The lid has depth ky cm, where  $0 < k \le 1$  (see diagram).

- (i) Use differentiation to find, in terms of k, the value of x which gives a minimum total external surface area of the box and the lid.
- (ii) Find also the ratio of the height to the width,  $\frac{y}{x}$ , in this case, simplifying your answer. [2]
- (iii) Find the values between which  $\frac{y}{x}$  must lie.
- (iv) Find the value of k for which the box has square ends.

10 The line *l* has equation 
$$\frac{x-10}{-3} = \frac{y+1}{6} = \frac{z+3}{9}$$
, and the plane *p* has equation  $x - 2y - 3z = 0$ .

- (i) Show that l is perpendicular to p. [2]
- (ii) Find the coordinates of the point of intersection of l and p.
- (iii) Show that the point A with coordinates (-2, 23, 33) lies on l. Find the coordinates of the point B which is the mirror image of A in p. [3]
- (iv) Find the area of triangle *OAB*, where *O* is the origin, giving your answer to the nearest whole number. [3]
- 11 A curve C has parametric equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}.$$

(i) The point P on the curve has parameter p. Show that the equation of the tangent at P is

$$(p^2 + 1)x - (p^2 - 1)y = 4p.$$
[4]

[2]

[2]

[4]

- (ii) The tangent at P meets the line y = x at the point A and the line y = -x at the point B. Show that the area of triangle OAB is independent of p, where O is the origin. [4]
- (iii) Find a cartesian equation of C. Sketch C, giving the coordinates of any points where C crosses the x- and y-axes and the equations of any asymptotes.

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## MATHEMATICS

Paper 2

9740/02

3 hours

October/November 2010

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#### Section A: Pure Mathematics [40 marks]

- 1 (i) Solve the equation  $x^2 6x + 34 = 0$ .
  - (ii) One root of the equation  $x^4 + 4x^3 + x^2 + ax + b = 0$ , where a and b are real, is x = -2 + i. Find the values of a and b and the other roots. [5]

2 (i) Prove by mathematical induction that 
$$\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7).$$
 [5]

(ii) (a) Prove by the method of differences that  $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}.$  [4]

(b) Explain why 
$$\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$$
 is a convergent series, and state the value of the sum to infinity. [2]

- 3 (i) Given that  $y = x\sqrt{(x+2)}$ , find  $\frac{dy}{dx}$ , expressing your answer as a single algebraic fraction. Hence show that there is only one value of x for which the curve  $y = x\sqrt{(x+2)}$  has a turning point, and state this value. [5]
  - (ii) A curve has equation  $y^2 = x^2(x+2)$ .
    - (a) Find exactly the possible values of the gradient at the point where x = 0. [2]
    - (b) Sketch the curve  $y^2 = x^2(x+2)$ . [2]
    - (iii) On a separate diagram sketch the graph of y = f'(x), where  $f(x) = x\sqrt{(x+2)}$ . State the equations of any asymptotes. [2]
- 4 The function f is defined as follows.

$$f: x \mapsto \frac{1}{x^2 - 1}$$
 for  $x \in \mathbb{R}$ ,  $x \neq -1$ ,  $x \neq 1$ .

- (i) Sketch the graph of y = f(x).
- (ii) If the domain of f is further restricted to  $x \ge k$ , state with a reason the least value of k for which the function  $f^{-1}$  exists. [2]

In the rest of the question, the domain of f is  $x \in \mathbb{R}$ ,  $x \neq -1$ ,  $x \neq 1$ , as originally defined.

The function g is defined as follows.

$$g: x \mapsto \frac{1}{x-3}$$
 for  $x \in \mathbb{R}$ ,  $x \neq 2$ ,  $x \neq 3$ ,  $x \neq 4$ .

(iii) Show that 
$$fg(x) = \frac{(x-3)^2}{(4-x)(x-2)}$$
. [2]

- (iv) Solve the inequality fg(x) > 0. [3]
- (v) Find the range of fg. [3]

[1]

#### Section B: Statistics [60 marks]

- At an international athletics competition, it is desired to sample 1% of the spectators to find their opinions of the catering facilities.
  - (i) Give a reason why it would be difficult to use a stratified sample. [1]
  - (ii) Explain how a systematic sample could be carried out. [2]
- $\odot$  6 The time required by an employee to complete a task is a normally distributed random variable. Over a long period it is known that the mean time required is 42.0 minutes. Background music is introduced in the workplace, and afterwards the time required, *t* minutes, is measured for a random sample of 11 employees. The results are summarised as follows.

$$n = 11$$
  $\Sigma t = 454.3$   $\Sigma t^2 = 18778.43$ 

Find unbiased estimates of the population mean and variance. Test, at the 10% significance level, whether there has been a change in the mean time required by an employee to complete the task. [7]

7 For events A and B it is given that P(A) = 0.7, P(B) = 0.6 and  $P(A \mid B') = 0.8$ . Find

(i) $P(A \cap B')$ ,	[2	]
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- (ii)  $P(A \cup B)$ , [2]
- (iii) P(B' | A). [2]

For a third event C, it is given that P(C) = 0.5 and that A and C are independent.

- (iv) Find  $P(A' \cap C)$ . [2]
- (v) Hence state an inequality satisfied by  $P(A' \cap B \cap C)$ . [1]
- 8 The digits 1, 2, 3, 4 and 5 are arranged randomly to form a five-digit number. No digit is repeated. Find the probability that

(i)	the number is greater than 30 000,	[1]
(ii)	the last two digits are both even,	[2]
(iii)	the number is greater than 30 000 and odd.	[4]

**9** In this question you should state clearly the values of the parameters of any normal distribution you use.

Over a three-month period Ken makes X minutes of peak-rate telephone calls and Y minutes of cheap-rate calls. X and Y are independent random variables with the distributions  $N(180, 30^2)$  and  $N(400, 60^2)$  respectively.

 (i) Find the probability that, over a three-month period, the number of minutes of cheap-rate calls made by Ken is more than twice the number of minutes of peak-rate calls.

Peak-rate calls cost \$0.12 per minute and cheap-rate calls cost \$0.05 per minute.

- (ii) Find the probability that, over a three-month period, the total cost of Ken's calls is greater than \$45.
- (iii) Find the probability that the total cost of Ken's peak-rate calls over two independent three-month periods is greater than \$45.
   [3]
- 10 A car is placed in a wind tunnel and the drag force F for different wind speeds v, in appropriate units, is recorded. The results are shown in the table.

v	0	4	8	12	16	20	24	28	32	36	
F	0	2.5	5.1	8.8	11.2	13.6	17.6	22.0	27.8	33.9	

(i) Draw the scatter diagram for these values, labelling the axes clearly.

It is thought that the drag force F can be modelled by one of the formulae

$$F = a + bv$$
 or  $F = c + dv^2$ 

where a, b, c and d are constants.

- (ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between
  - (a) v and F,
  - (b)  $v^2$  and F.
- (iii) Use your answers to parts (i) and (ii) to explain which of F = a + bv or  $F = c + dv^2$  is the better model. [1]
- (iv) It is required to estimate the value of v for which F = 26.0. Find the equation of a suitable regression line, and use it to find the required estimate. Explain why neither the regression line of v on F nor the regression line of  $v^2$  on F should be used. [4]

[2]

[2]

In this question you should state clearly all distributions that you use, together with the values of the appropriate parameters.

The number of telephone calls received by a call centre in one minute is a random variable with the distribution Po(3).

- (i) Find the probability that exactly 8 calls are received in a randomly chosen period of 4 minutes.
- (ii) Find the length of time, to the nearest second, for which the probability that no calls are received is 0.2.
- (iii) Use a suitable approximation to find the probability that, on a randomly chosen working day of 12 hours, more than 2200 calls are received. [4]

A working day of 12 hours on which more than 2200 calls are received is said to be 'busy'.

- (iv) Find the probability that, in six randomly chosen working days, exactly two are busy. [2]
- (v) Use a suitable approximation to find the probability that, in 30 randomly chosen working days of 12 hours, fewer than 10 are busy.

[2]

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