MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/01

Paper 1

October/November 2010

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 The position vectors **a** and **b** are given by

$$\mathbf{a} = 2p\mathbf{i} + 3p\mathbf{j} + 6p\mathbf{k} \quad \text{and} \quad \mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k},$$

where $p > 0$. It is given that $|\mathbf{a}| = |\mathbf{b}|$.

(i) Find the exact value of p. [2]

(ii) Show that $(\mathbf{a} + \mathbf{b}).(\mathbf{a} - \mathbf{b}) = 0$. [3]

2 **(i)** Find the first three terms of the Maclaurin series for $e^x(1 + \sin 2x)$. [You may use standard results given in the List of Formulae (MF15).] [3]

(ii) It is given that the first two terms of this series are equal to the first two terms in the series expansion, in ascending powers of x, of $\left(1 + \frac{4}{3}x\right)^n$. Find n and show that the third terms in each of these series are equal. [3]

3 The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n = n(2n + c),$$

where c is a constant.

(i) Find u_n in terms of c and n. [3]

⑤ **(ii)** Find a recurrence relation of the form $u_{n+1} = f(u_n)$. [2]

4 **(i)** Given that $x^2 - y^2 + 2xy + 4 = 0$, find $\dfrac{dy}{dx}$ in terms of x and y. [4]

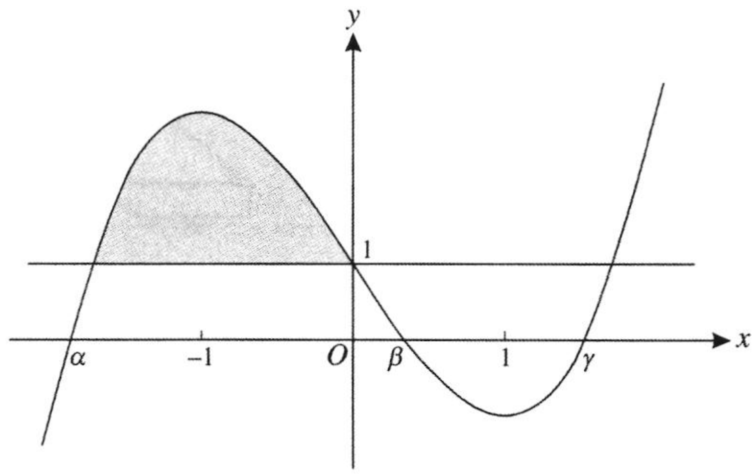
(ii) For the curve with equation $x^2 - y^2 + 2xy + 4 = 0$, find the coordinates of each point at which the tangent is parallel to the x-axis. [4]

5 The curve with equation $y = x^3$ is transformed by a translation of 2 units in the positive x-direction, followed by a stretch with scale factor $\frac{1}{2}$ parallel to the y-axis, followed by a translation of 6 units in the negative y-direction.

(i) Find the equation of the new curve in the form $y = f(x)$ and the exact coordinates of the points where this curve crosses the x- and y-axes. Sketch the new curve. [5]

(ii) On the same diagram, sketch the graph of $y = f^{-1}(x)$, stating the exact coordinates of the points where the graph crosses the x- and y-axes. [3]

6



The diagram shows the curve with equation $y = x^3 - 3x + 1$ and the line with equation $y = 1$. The curve crosses the x-axis at $x = \alpha$, $x = \beta$ and $x = \gamma$ and has turning points at $x = -1$ and $x = 1$.

(i) Find the values of β and γ, giving your answers correct to 3 decimal places. [2]

(ii) Find the area of the region bounded by the curve and the x-axis between $x = \beta$ and $x = \gamma$. [2]

(iii) Use a non-calculator method to find the area of the shaded region between the curve and the line. [4]

(iv) Find the set of values of k for which the equation $x^3 - 3x + 1 = k$ has three real distinct roots. [2]

7 A bottle containing liquid is taken from a refrigerator and placed in a room where the temperature is a constant 20 °C. As the liquid warms up, the rate of increase of its temperature θ °C after time t minutes is proportional to the temperature difference $(20 - \theta)$ °C. Initially the temperature of the liquid is 10 °C and the rate of increase of the temperature is 1 °C per minute. By setting up and solving a differential equation, show that $\theta = 20 - 10e^{-\frac{1}{10}t}$. [7]

Find the time it takes the liquid to reach a temperature of 15 °C, and state what happens to θ for large values of t. Sketch a graph of θ against t. [4]

8 The complex numbers z_1 and z_2 are given by $1 + i\sqrt{3}$ and $-1 - i$ respectively.

(i) Express each of z_1 and z_2 in polar form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give r and θ in exact form. [2]

(ii) Find the complex conjugate of $\dfrac{z_1}{z_2}$ in exact polar form. [3]

(iii) On a single Argand diagram, sketch the loci

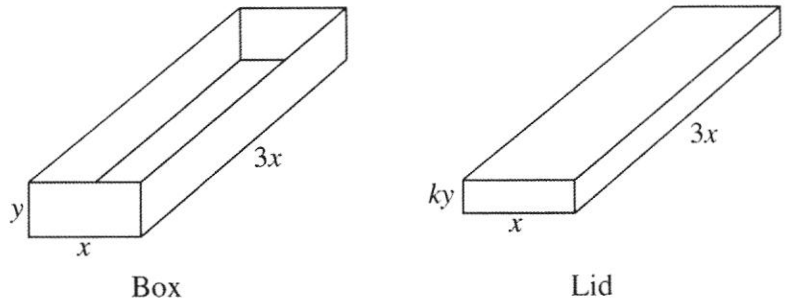
(a) $|z - z_1| = 2$,

(b) $\arg(z - z_2) = \frac{1}{4}\pi$.

[4]

(iv) Find where the locus $|z - z_1| = 2$ meets the positive real axis. [2]

9



A company requires a box made of cardboard of negligible thickness to hold 300 cm^3 of powder when full. The length of the box is $3x$ cm, the width is x cm and the height is y cm. The lid has depth ky cm, where $0 < k \leqslant 1$ (see diagram).

(i) Use differentiation to find, in terms of k, the value of x which gives a minimum total external surface area of the box and the lid. [6]

(ii) Find also the ratio of the height to the width, $\frac{y}{x}$, in this case, simplifying your answer. [2]

(iii) Find the values between which $\frac{y}{x}$ must lie. [2]

(iv) Find the value of k for which the box has square ends. [2]

10 The line l has equation $\frac{x-10}{-3} = \frac{y+1}{6} = \frac{z+3}{9}$, and the plane p has equation $x - 2y - 3z = 0$.

(i) Show that l is perpendicular to p. [2]

(ii) Find the coordinates of the point of intersection of l and p. [4]

(iii) Show that the point A with coordinates $(-2, 23, 33)$ lies on l. Find the coordinates of the point B which is the mirror image of A in p. [3]

(iv) Find the area of triangle OAB, where O is the origin, giving your answer to the nearest whole number. [3]

11 A curve C has parametric equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}.$$

(i) The point P on the curve has parameter p. Show that the equation of the tangent at P is

$$(p^2 + 1)x - (p^2 - 1)y = 4p.$$ [4]

(ii) The tangent at P meets the line $y = x$ at the point A and the line $y = -x$ at the point B. Show that the area of triangle OAB is independent of p, where O is the origin. [4]

(iii) Find a cartesian equation of C. Sketch C, giving the coordinates of any points where C crosses the x- and y-axes and the equations of any asymptotes. [4]