

MATHEMATICS

9740/01

Paper 1

October/November 2015

3 hours

Additional Materials: Answer Paper
 Graph paper
 List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

- 1 A curve C has equation

$$y = \frac{a}{x^2} + bx + c,$$

where a , b and c are constants. It is given that C passes through the points with coordinates $(1.6, -2.4)$ and $(-0.7, 3.6)$, and that the gradient of C is 2 at the point where $x = 1$.

- (i) Find the values of a , b and c , giving your answers correct to 3 decimal places. [4]
- (ii) Find the x -coordinate of the point where C crosses the x -axis, giving your answer correct to 3 decimal places. [2]
- (iii) One asymptote of C is the line with equation $x = 0$. Write down the equation of the other asymptote of C . [1]
- 2 (i) Sketch the curve with equation $y = \left| \frac{x+1}{1-x} \right|$, stating the equations of the asymptotes. On the same diagram, sketch the line with equation $y = x + 2$. [3]
- (ii) Solve the inequality $\left| \frac{x+1}{1-x} \right| < x + 2$. [3]

- 3 (i) Given that f is a continuous function, explain, with the aid of a sketch, why the value of

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f\left(\frac{n}{n}\right) \right\}$$

is $\int_0^1 f(x) dx$. [2]

- (ii) Hence evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{\sqrt[3]{1} + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{\sqrt[3]{n}} \right)$. [3]

- 4 A piece of wire of fixed length d m is cut into two parts. One part is bent into the shape of a rectangle with sides of length x m and y m. The other part is bent into the shape of a semicircle, including its diameter. The radius of the semicircle is x m. Show that the maximum value of the total area of the two shapes can be expressed as $kd^2 \text{ m}^2$, where k is a constant to be found. [6]

- 5 (i) State a sequence of transformations that will transform the curve with equation $y = x^2$ on to the curve with equation $y = \frac{1}{4}(x-3)^2$. [2]

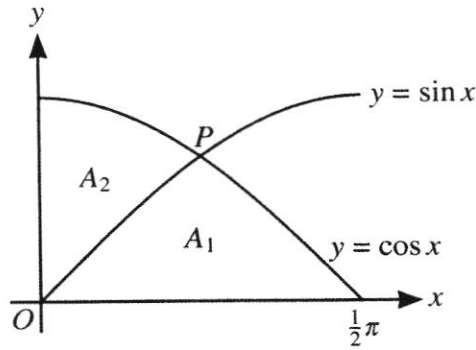
A curve has equation $y = f(x)$, where

$$f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1, \\ \frac{1}{4}(x-3)^2 & \text{for } 1 < x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

- (ii) Sketch the curve for $-1 \leq x \leq 4$. [3]
- (iii) On a separate diagram, sketch the curve with equation $y = 1 + f\left(\frac{1}{2}x\right)$, for $-1 \leq x \leq 4$. [2]

- 6 (i) Write down the first three non-zero terms in the Maclaurin series for $\ln(1 + 2x)$, where $-\frac{1}{2} < x \leq \frac{1}{2}$, simplifying the coefficients. [2]
- (ii) It is given that the three terms found in part (i) are equal to the first three terms in the series expansion of $ax(1 + bx)^c$ for small x . Find the exact values of the constants a , b and c and use these values to find the coefficient of x^4 in the expansion of $ax(1 + bx)^c$, giving your answer as a simplified rational number. [6]
- 7 Referred to the origin O , points A and B have position vectors \mathbf{a} and \mathbf{b} respectively. Point C lies on OA , between O and A , such that $OC : CA = 3 : 2$. Point D lies on OB , between O and B , such that $OD : DB = 5 : 6$.
- (i) Find the position vectors \overrightarrow{OC} and \overrightarrow{OD} , giving your answers in terms of \mathbf{a} and \mathbf{b} . [2]
- (ii) Show that the vector equation of the line BC can be written as $\mathbf{r} = \frac{3}{5}\lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$, where λ is a parameter. Find in a similar form the vector equation of the line AD in terms of a parameter μ . [3]
- (iii) Find, in terms of \mathbf{a} and \mathbf{b} , the position vector of the point E where the lines BC and AD meet and find the ratio $AE : ED$. [5]
- 8 Two athletes are to run 20 km by running 50 laps around a circular track of length 400 m. They aim to complete the distance in between $1\frac{1}{2}$ hours and $1\frac{3}{4}$ hours inclusive.
- (i) Athlete A runs the first lap in T seconds and each subsequent lap takes 2 seconds longer than the previous lap. Find the set of values of T which will enable A to complete the distance within the required time interval. [4]
- (ii) Athlete B runs the first lap in t seconds and the time for each subsequent lap is 2% more than the time for the previous lap. Find the set of values of t which will enable B to complete the distance within the required time interval. [4]
- (iii) Assuming each athlete completes the 20 km run in exactly $1\frac{1}{2}$ hours, find the difference in the athletes' times for their 50th laps, giving your answer to the nearest second. [3]
- 9 (a) The complex number w is such that $w = a + ib$, where a and b are non-zero real numbers. The complex conjugate of w is denoted by w^* . Given that $\frac{w^2}{w^*}$ is purely imaginary, find the possible values of w in terms of a . [5]
- ⊕ (b) The complex number z is such that $z^5 = -32i$.
- (i) Find the modulus and argument of each of the possible values of z . [4]
- (ii) Two of these values are z_1 and z_2 , where $\frac{1}{2}\pi < \arg z_1 < \pi$ and $-\pi < \arg z_2 < -\frac{1}{2}\pi$. Find the exact value of $\arg(z_1 - z_2)$ in terms of π and show that $|z_1 - z_2| = 4\sin(\frac{1}{5}\pi)$. [4]

10 Do not use a calculator in answering this question.



With origin O , the curves with equations $y = \sin x$ and $y = \cos x$, where $0 \leq x \leq \frac{1}{2}\pi$, meet at the point P with coordinates $(\frac{1}{4}\pi, \frac{1}{2}\sqrt{2})$. The area of the region bounded by the curves and the x -axis is A_1 and the area of the region bounded by the curves and the y -axis is A_2 (see diagram).

(i) Show that $\frac{A_1}{A_2} = \sqrt{2}$. [4]

(ii) The region bounded by $y = \sin x$ between O and P , the line $y = \frac{1}{2}\sqrt{2}$ and the y -axis is rotated about the y -axis through 360° . Show that the volume of the solid formed is given by

$$\pi \int_0^{\frac{1}{2}\sqrt{2}} (\sin^{-1} y)^2 dy. \quad [2]$$

(iii) Show that the substitution $y = \sin u$ transforms the integral in part (ii) to $\pi \int_a^b u^2 \cos u du$, for limits a and b to be determined. Hence find the exact volume. [6]

11 A curve C has parametric equations

$$x = \sin^3 \theta, \quad y = 3 \sin^2 \theta \cos \theta, \quad \text{for } 0 \leq \theta \leq \frac{1}{2}\pi.$$

(i) Show that $\frac{dy}{dx} = 2 \cot \theta - \tan \theta$. [3]

(ii) Show that C has a turning point when $\tan \theta = \sqrt{k}$, where k is an integer to be determined. Find, in non-trigonometric form, the exact coordinates of the turning point and explain why it is a maximum. [6]

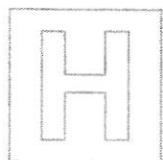
(iii) Show that the area of the region bounded by C and the x -axis is given by

$$\int_0^{\frac{1}{2}\pi} 9 \sin^4 \theta \cos^2 \theta d\theta.$$

Use your calculator to find the area, giving your answer correct to 3 decimal places. [3]

The line with equation $y = ax$, where a is a positive constant, meets C at the origin and at the point P .

(iv) Show that $\tan \theta = \frac{3}{a}$ at P . Find the exact value of a such that the line passes through the maximum point of C . [3]



MATHEMATICS

9740/02

Paper 2

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Section A: Pure Mathematics [40 marks]

- 1 As a tree grows, the rate of increase of its height, h m, with respect to time, t years after planting, is modelled by the differential equation

$$\frac{dh}{dt} = \frac{1}{10}\sqrt{(16 - \frac{1}{2}h)}.$$

The tree is planted as a seedling of negligible height, so that $h = 0$ when $t = 0$.

- (i) State the maximum height of the tree, according to this model. [1]
- (ii) Find an expression for t in terms of h , and hence find the time the tree takes to reach half its maximum height. [5]

- 2 The line L has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 4\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$.

- (i) Find the acute angle between L and the x -axis. [2]

The point P has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$.

- (ii) Find the points on L which are a distance of $\sqrt{(33)}$ from P . Hence or otherwise find the point on L which is closest to P . [5]
- (iii) Find a cartesian equation of the plane that includes the line L and the point P . [3]

- 3 (a) The function f is defined by $f : x \mapsto \frac{1}{1-x^2}$, $x \in \mathbb{R}$, $x > 1$.

- (i) Show that f has an inverse. [2]

- (ii) Find $f^{-1}(x)$ and state the domain of f^{-1} . [3]

- (b) The function g is defined by $g : x \mapsto \frac{2+x}{1-x^2}$, $x \in \mathbb{R}$, $x \neq \pm 1$. Find algebraically the range of g , giving your answer in terms of $\sqrt{3}$ as simply as possible. [5]

- 4 (a) Prove by the method of mathematical induction that

$$1 \times 3 \times 6 + 2 \times 4 \times 7 + 3 \times 5 \times 8 + \dots + n(n+2)(n+5) = \frac{1}{12}n(n+1)(3n^2 + 31n + 74). \quad [6]$$

- (b) (i) Show that $\frac{2}{4r^2 + 8r + 3}$ can be expressed as $\frac{A}{2r+1} + \frac{B}{2r+3}$, where A and B are constants to be determined. [1]

The sum $\sum_{r=1}^n \frac{2}{4r^2 + 8r + 3}$ is denoted by S_n .

- (ii) Find an expression for S_n in terms of n . [4]
- (iii) Find the smallest value of n for which S_n is within 10^{-3} of the sum to infinity. [3]

Section B: Statistics [60 marks]

- ⑤ The manager of a busy supermarket wishes to conduct a survey of the opinions of customers of different ages about different types of cola drink.
- (i) Give a reason why the manager would not be able to use stratified sampling. [1]
 - (ii) Explain briefly how the manager could carry out a survey using quota sampling. [2]
 - (iii) Give one reason why quota sampling would not necessarily provide a sample which is representative of the customers of the supermarket. [1]
- 6 'Droppers' are small sweets that are made in a variety of colours. Droppers are sold in packets and the colours of the sweets in a packet are independent of each other. On average, 25% of Droppers are red.
- (i) A small packet of Droppers contains 10 sweets. Find the probability that there are at least 4 red sweets in a small packet. [2]
- A large packet of Droppers contains 100 sweets.
- ⑥ (ii) Use a suitable approximation, which should be stated, to find the probability that a large packet contains at least 30 red sweets. [3]
- (iii) Yip buys 15 large packets of Droppers. Find the probability that no more than 3 of these packets contain at least 30 red sweets. [2]
- ⑦ The average number of errors per page for a certain daily newspaper is being investigated.
- (i) State, in context, two assumptions that need to be made for the number of errors per page to be well modelled by a Poisson distribution. [2]
- Assume that the number of errors per page has the distribution $Po(1.3)$.
- (ii) Find the probability that, on one day, there are more than 10 errors altogether on the first 6 pages. [3]
 - (iii) The probability that there are fewer than 2 errors altogether on the first n pages of the newspaper is less than 0.05. Write down an inequality in terms of n to represent this information, and hence find the least possible value of n . [2]
- ⑧ A market stall sells pineapples which have masses that are normally distributed. The stall owner claims that the mean mass of the pineapples is at least 0.9 kg. Nur buys a random selection of 8 pineapples from the stall. The 8 pineapples have masses, in kg, as follows.
- 0.80 1.00 0.82 0.85 0.93 0.96 0.81 0.89
- Find unbiased estimates of the population mean and variance of the mass of pineapples. Test at the 10% level of significance whether there is any evidence to doubt the stall owner's claim. [7]

- 9 For events A , B and C it is given that $P(A) = 0.45$, $P(B) = 0.4$, $P(C) = 0.3$ and $P(A \cap B \cap C) = 0.1$. It is also given that events A and B are independent, and that events A and C are independent.

- (i) Find $P(B | A)$. [1]
- (ii) Given also that events B and C are independent, find $P(A' \cap B' \cap C')$. [3]
- (iii) Given instead that events B and C are **not** independent, find the greatest and least possible values of $P(A' \cap B' \cap C')$. [4]

- 10 In an experiment the following information was gathered about air pressure P , measured in inches of mercury, at different heights above sea-level h , measured in feet.

h	2000	5000	10 000	15 000	20 000	25 000	30 000	35 000	40 000	45 000
P	27.8	24.9	20.6	16.9	13.8	11.1	8.89	7.04	5.52	4.28

- (i) Draw a scatter diagram for these values, labelling the axes. [1]
- (ii) Find, correct to 4 decimal places, the product moment correlation coefficient between
- (a) h and P ,
- (b) $\ln h$ and P ,
- (c) \sqrt{h} and P . [3]
- (iii) Using the most appropriate case from part (ii), find the equation which best models air pressure at different heights. [2]
- (iv) Given that 1 metre = 3.28 feet, re-write your equation from part (iii) so that it can be used to estimate the air pressure when the height is given in metres. [2]
- 11 This question is about arrangements of all eight letters in the word CABBAGES.
- (i) Find the number of different arrangements of the eight letters that can be made. [2]
- (ii) Write down the number of these arrangements in which the letters are **not** in alphabetical order. [1]
- (iii) Find the number of different arrangements that can be made with both the A's together and both the B's together. [2]
- (iv) Find the number of different arrangements that can be made with no two adjacent letters the same. [4]

- 12** In this question you should state clearly the values of the parameters of any normal distribution you use.

The masses in grams of apples have the distribution $N(300, 20^2)$ and the masses in grams of pears have the distribution $N(200, 15^2)$. A certain recipe requires 5 apples and 8 pears.

- (i) Find the probability that the total mass of 5 randomly chosen apples is more than 1600 grams. [2]
- (ii) Find the probability that the total mass of 5 randomly chosen apples is more than the total mass of 8 randomly chosen pears. [3]

The recipe requires the apples and pears to be prepared by peeling them and removing the cores. This process reduces the mass of each apple by 15% and the mass of each pear by 10%.

- (iii) Find the probability that the total mass, after preparation, of 5 randomly chosen apples and 8 randomly chosen pears is less than 2750 grams. [4]

