2017 H2 MA Prelim Compilation - Complex Numbers (29 Questions)

ACJC Prelim 9758/2017/01/Q7

(a) Given that $2z+1=|w|$ and $2w-z=4+8\mathrm{i}$, solve for w and z. [5]

(b) Find the exact values of x and y, where $x, y \in \mathbb{R}$, such that $2\mathrm{e}^{-\left(\frac{3+x+\mathrm{i}y}{\mathrm{i}}\right)}=1-\mathrm{i}$. [4]

Answers

(a) $z=2,\quad w=3+4\mathrm{i}$; (b) $x=-\dfrac{\pi}{4}-3,\ y=\dfrac{1}{2}\ln 2$.

ACJC Prelim 9758/2017/02/Q1

Given that $1+\mathrm{i}$ is a root of the equation $z^3-4(1+\mathrm{i})z^2+(-2+9\mathrm{i})z+5-\mathrm{i}=0$, find the other roots of the equation. [4]

Answers

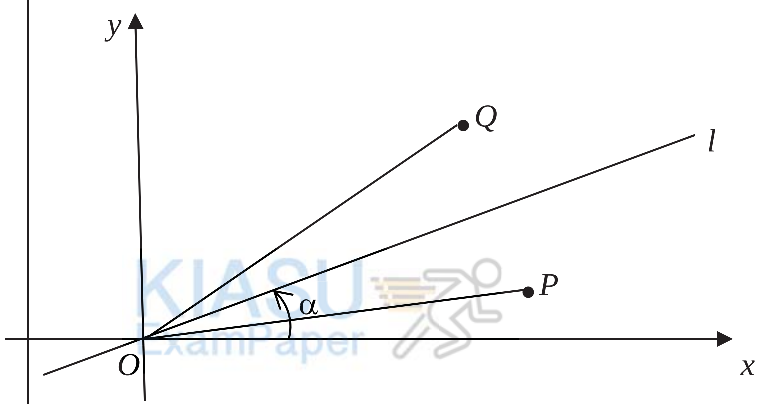
$z=3+2\mathrm{i}$ or $z=\mathrm{i}$

AJC Prelim 9758/2017/01/Q6

The diagram below shows the line l that passes through the origin and makes an angle α with the positive real axis, where $0<\alpha<\dfrac{\pi}{2}$.

Point P represents the complex number z_1 where $0<\arg z_1<\alpha$ and length of OP is r units.

Point P is reflected in line l to produce point Q, which represents the complex number z_2.



Prove that $\arg z_1 + \arg z_2 = 2\alpha$. [2]

	Deduce that $z_1 z_2 = r^2(\cos 2\alpha + i\sin 2\alpha)$. [1] Let R be the point that represents the complex number $z_1 z_2$. Given that $\alpha = \frac{\pi}{4}$, write down the cartesian equation of the locus of R as z_1 varies. [2]
	Answers $x = 0, y > 0$

	AJC Prelim 9758/2017/02/Q2
	The polynomial P(z) has real coefficients. The equation P(z) = 0 has a root $re^{i\theta}$, where $r > 0$ and $0 < \theta < \pi$. Write down a second root in terms of r and θ, and hence show that a quadratic factor of P(z) is $z^2 - 2rz\cos\theta + r^2$. [2] Let P(z) = $z^3 + az^2 + 15z + 18$ where a is a real number. One of the roots of the equation P(z) = 0 is $3e^{i\left(\frac{2\pi}{3}\right)}$. By expressing P(z) as a product of two factors with real coefficients, find a and the other roots of P(z) = 0. [4] Deduce the roots of the equation $18z^3 + 15z^2 + az + 1 = 0$. [2]
	Answers $a = 5$ $3e^{i\left(\frac{2\pi}{3}\right)}$, $3e^{i\left(-\frac{2\pi}{3}\right)}$ and $-2 = 2e^{i(\pi)}$ $z = \frac{1}{3}e^{i\left(-\frac{2\pi}{3}\right)}, \frac{1}{3}e^{i\left(\frac{2\pi}{3}\right)}, -\frac{1}{2}$

CJC Prelim 9758/2017/02/Q4
(a) The complex numbers z and w satisfy the simultaneous equations $$z + w^* + 5i = 10 \quad \text{and} \quad \lvert w\rvert^2 = z + 18 + i.$$ Find z and w. [4] **(b)** **(i)** It is given that $2+i$ is a root of the equation $z^2 - 5z + 7 + i = 0$. Find the second root of the equation in cartesian form, showing your working clearly. [2] **(ii)** Hence find the roots of the equation $-iw^2 + 5w + 7i - 1 = 0$. [2]

(c) The complex number z is given by $z=-a+a\mathrm{i}$, where a is a positive real number.

(i) It is given that $w=-\frac{\sqrt{2}z^*}{z^4}$. Express w in the form $re^{\mathrm{i}\theta}$, in terms of a, where $r>0$ and $-\pi<\theta\leq\pi$.

[4]

(ii) Find the two smallest positive whole number values of n such that $\mathrm{Re}\,(w^n)=0$.

[3]

Answers

(a) $w=3+4\mathrm{i},\ z=7-\mathrm{i}$

$w=-4+4\mathrm{i},\ z=14-\mathrm{i}$

(b)(i) $3-\mathrm{i}$

(ii) $w=1-2\mathrm{i},\ w=-1-3\mathrm{i}$

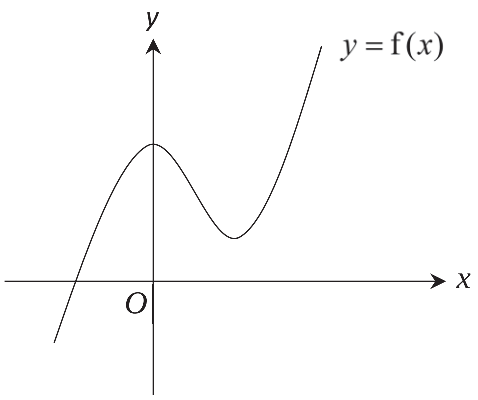
(c)(i) $\frac{1}{2a^3}e^{\mathrm{i}\left(-\frac{3\pi}{4}\right)}$

(ii) 2, 6

DHS Prelim 9758/2017/01/Q8

Do not use a graphic calculator in answering this question.

(a)



It is given that $\mathrm{f}(x)$ is a cubic polynomial with real coefficients. The diagram shows the curve with equation $y=\mathrm{f}(x)$. What can be said about all the roots of the equation $\mathrm{f}(x)=0$?

[2]

(b) The equation $2z^2-(7+6\mathrm{i})z+11+\mathrm{i}c=0$, where c is a non-zero real number, has a root $z=3+4\mathrm{i}$. Show that $c=-2$. Determine the other root of the equation in cartesian form. Hence find the roots of the equation $2w^2+\left(-6+7\mathrm{i}\right)w-11+2\mathrm{i}=0$. [6]

(c) The complex number z is given by $z = 1 + e^{i\alpha}$.

(i) Show that z can be expressed as $2\cos\left(\frac{1}{2}\alpha\right)e^{i\left(\frac{1}{2}\alpha\right)}$. [2]

(ii) Given $\alpha = \frac{1}{3}\pi$ and $w = -1 - \sqrt{3}i$, find the exact modulus and argument of $\left(\dfrac{z}{w^3}\right)^*$. [5]

Answers

(a) Since the curve shows only one x-intercept, it means that there is only one real root in the equation $f(x) = 0$.

Since the equation has all real coefficients, then the two other roots must be non-real and they are a conjugate pair.

(b) $\frac{1}{2} - i$; $4 - 3i$ and $-1 - \frac{1}{2}i$.

(c) (ii) $\frac{\sqrt{3}}{8}$; $-\frac{\pi}{6}$

HCI Prelim 9758/2017/01/Q4

The complex number z is given by $z = re^{i\theta}$, where $r > 0$ and $0 \le \theta \le \pi$. It is given that the complex number $w = \left(-\sqrt{3} - i\right)z$.

(i) Find $|w|$ in terms of r, and $\arg w$ in terms of θ. [2]

(ii) Given that $\dfrac{z^8}{w^*}$ is purely imaginary, find the three smallest values of θ in terms of π. [5]

Answers

(i) $|w| = 2r$, $\arg w = -\dfrac{5\pi}{6} + \theta$

(ii) $9\theta - \dfrac{5\pi}{6} = \ldots, -\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

The three smallest values of θ are $\dfrac{\pi}{27}$, $\dfrac{4\pi}{27}$ and $\dfrac{7\pi}{27}$.

HCI Prelim 9758/2017/02/Q2

The complex numbers z and w satisfy the following equations $$2z+3w=20\ ,$$ $$w-zw^*=6+22\mathrm{i}\ .$$ **(i)** Find z and w in the form $a+b\mathrm{i}$, where a and b are real, $a\neq 0$. [5]   **(ii)** Show z and w on a single Argand diagram, indicating clearly their modulus. State the relationship between z and w with reference to the origin O. [2]
Answers   (i) $w=6+2\mathrm{i},\ z=1-3\mathrm{i}$ (ii) $\angle WOZ$ is $90°$

	IJC Prelim 9758/2017/01/Q4
	A graphic calculator is **not** to be used in answering this question. **(a)** The equation $w^3+pw^2+qw+30=0$, where p and q are real constants, has a root $w=2-\mathrm{i}$. Find the values of p and q, showing your working. [3] **(b)** The equation $z^2+(-5+2\mathrm{i})z+(21-\mathrm{i})=0$ has a root $z=3+u\mathrm{i}$, where u is real constant. Find the value of u and hence find the second root of the equation in cartesian form, $a+b\mathrm{i}$, showing your working. [5]
	Answers (a) $p=2, q=-19$ (b) $u=-5$, $z=2+3\mathrm{i}$

	IJC Prelim 9758/2017/02/Q1
	The complex number z is such that $\lvert z\rvert=1$ and $\arg z=\theta$, where $0<\theta<\frac{\pi}{4}$. **(i)** Mark a possible point A representing z on an Argand diagram. Hence, mark the points B and C representing z^2 and $z+z^2$ respectively on the same Argand diagram corresponding to point A. [2] **(ii)** State the geometrical shape of $OACB$. [1] **(iii)** Express $z+z^2$ in polar form, $p\cos(q\theta)\left[\cos(k\theta)+\mathrm{i}\sin(k\theta)\right]$, where p, q and k are constants to be determined. [2]

	Answers (ii) rhombus (iii) $2\cos\frac{\theta}{2}\left[\cos\frac{3\theta}{2}+\mathrm{i}\sin\frac{3\theta}{2}\right]$

JJC Prelim 9758/2017/01/Q7
(a) If $u=2-\mathrm{i}\sin^2\theta$ and $v=2\cos^2\theta+\mathrm{i}\sin^2\theta$ where $-\pi<\theta\le\pi$, find $u-v$ in terms of $\sin^2\theta$, and hence determine the exact expression for $\lvert u-v\rvert$ and the exact value of $\arg(u-v)$. [6] **(b)** The roots of the equation $x^2+(\mathrm{i}-3)x+2(1-\mathrm{i})=0$ are α and β, where α is a real number and β is not a real number. Find α and β. [4]
Answers 7(a) $u-v=2\sin^2\theta-2\mathrm{i}\sin^2\theta$ $\lvert u-v\rvert=2\sqrt{2}\sin^2\theta$, $\arg(u-v)=-\frac{\pi}{4}$ 7(b) $\alpha=2$, $\beta=1-\mathrm{i}$

MI Prelim 9740/2017/01/Q10
(a) It is given that $-1+\mathrm{i}$ is a root of the equation $2z^3+az^2+bz+(3+\mathrm{i})=0$. **(i)** Find the values of the real numbers *a* and *b*. [4] **(ii)** Using these values of *a* and *b*, find the other roots of this equation. [3] **(b)** It is given that $w=-1+(\sqrt{3})\mathrm{i}$. **(i)** Without using a calculator, find an exact expression for w^5. Give your answer in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $0\le\theta\le2\pi$. [3] **(ii)** Without using a calculator, find the three smallest positive whole number values of *n* for which $\frac{w^*}{w^n}$ is a real number. [4]

Answers

(a)(i) $a=6, b=7$

(a)(ii) $z=-\frac{1}{2}-\frac{1}{2}\mathrm{i}$ or $z=-\frac{3}{2}-\frac{1}{2}\mathrm{i}$

(b)(i) $32\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}\right)}$

(b)(ii) 2, 5, 8.

MJC Prelim 9758/2017/01/Q3

Do not use a calculator in answering this question.

Showing your working, find the complex numbers z and w which satisfy the simultaneous equations

$$4\mathrm{i}z-3w=1+5\mathrm{i} \quad \text{and}$$

$$2z+(1+\mathrm{i})w=2+6\mathrm{i}.$$ [5]

Answers

$w=-3+5\mathrm{i}$ and $z=5+2\mathrm{i}$

MJC Prelim 9758/2017/02/Q1

The complex number z has modulus 3 and argument $\frac{2\pi}{3}$.

(i) Find the modulus and argument of $\frac{-2\mathrm{i}}{z^*}$, where z^* is the complex conjugate of z, leaving your answers in the exact form. [3]

(ii) Hence express $\frac{-2\mathrm{i}}{z^*}$ in the form of $x+\mathrm{i}y$, where x and y are real constants, giving the exact values of x and y in non-trigonometrical form. [2]

(iii) The complex number w is defined such that $w=1+\mathrm{i}k$, where k is a non-zero real constant. Given that $\frac{-2\mathrm{i}w}{z^*}$ is purely imaginary, find the exact value of k. [2]

Answers

(i) $\frac{2}{3}$; $\frac{\pi}{6}$

(ii) $\dfrac{\sqrt{3}}{3}+\dfrac{1}{3}\mathrm{i}$ (iii) $k=\sqrt{3}$

NJC Prelim 9758/2017/01/Q5

Do not use a calculator in answering this question.

(a) Showing your working clearly, find the complex numbers z and w which satisfy the simultaneous equations

$$\mathrm{i}z+w=2 \quad \text{and}$$
$$zw^*=2+4\mathrm{i},$$

where w^* is the complex conjugate of w. [5]

(b) The complex number p is given by $a+\mathrm{i}b$, where $a>0$, $b<0$, $a^2+b^2>1$ and $\tan^{-1}\left(\dfrac{b}{a}\right)=-\dfrac{2\pi}{9}$.

(i) Express the complex number $\dfrac{1}{p^2}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where r is in terms of a and b, and $-\pi<\theta\le\pi$. [2]

(ii) On a single Argand diagram, illustrate the points P and Q representing the complex numbers p and $\dfrac{1}{p^2}$ respectively, labelling clearly their modulus and argument. [2]

(iii) It is given that $\angle OPQ=\alpha$. Using sine rule, show that $|p|^3\approx\dfrac{\sqrt{3}}{2\alpha}-\dfrac{1}{2}-\dfrac{\alpha}{2\sqrt{3}}$ where α is small. [4]

Answers

$w=3-\mathrm{i},\ z=1+\mathrm{i}$;
$w=-1-\mathrm{i},\ z=1-3\mathrm{i}$

$\dfrac{1}{p^2}=\dfrac{1}{a^2+b^2}\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{9}\right)}$;

$\dfrac{\sqrt{3}}{2x}-\dfrac{1}{2}-\dfrac{1}{2\sqrt{3}}x$

	NYJC Prelim 9758/2017/01/Q3
	Do not use a calculator in answering this question.

	(i) Explain why the equation $z^3+az^2+az+7=0$ cannot have more than two non-real roots, where a is a real constant. [1] **(ii)** Given that $z=-7$ is a root of the equation in **(i)**, find the other roots, leaving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [4] **(iii)** Hence, solve the equation $\mathrm{i}z^3+8z^2-8\mathrm{i}z-7=0$, leaving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [2]
	Answers (i) Since a is real, the polynomial equation has real coefficients, and thus all non-real roots must be in conjugate pairs. Since the degree of the polynomial is three, there will be 3 roots. The highest even number below 3 is 2. (ii) $z=7e^{i\pi},\ \mathrm{e}^{\frac{i2\pi}{3}},\mathrm{e}^{-\frac{i2\pi}{3}}$ (iii) $z=7e^{\frac{i\pi}{2}},\ \mathrm{e}^{\frac{i\pi}{6}},\mathrm{e}^{\frac{i5\pi}{6}}$

PJC Prelim 9758/2017/02/Q4
Do not use a graphic calculator in answering this question. The complex number z is given by $z=-1+\mathrm{i}c$, where c is a non-zero real number. Given that $\frac{z^n}{z^*}$ is purely real, find **(i)** the possible values of c when $n=2$, [4] **(ii)** the three smallest positive integer values of n when $c=\sqrt{3}$. [5]
Answers (i) $c=0$, $c=\pm\sqrt{3}$ (ii) Three smallest positive integer values of n are $2,5,8$

RI Prelim 9758/2017/01/Q9
Do not use a calculator in answering this question. **(a)** One root of the equation $z^4+2z^3+az^2+bz+50=0$, where a and b are real, is $z=1+3\mathrm{i}$.

(i) Show that $a = 7$ and $b = 30$ and find the other roots of the equation. [5]

(ii) Deduce the roots of the equation $w^4 - 2\mathrm{i}w^3 - 7w^2 + 30\mathrm{i}w + 50 = 0$. [2]

(b) Given that $p^* = \dfrac{\left(-\dfrac{1}{\sqrt{3}} + \mathrm{i}\right)^5}{(1-\mathrm{i})^4}$, by considering the modulus and argument of p^*, find the exact expression for p, in cartesian form $x + \mathrm{i}y$. [4]

Answers

(a)(i) $z = 1 - 3\mathrm{i}$, $z = -2 + \mathrm{i}$ and $z = -2 - \mathrm{i}$.

(a)(ii) $w = -\mathrm{i} - 3$, $w = -\mathrm{i} + 3$, $w = 2\mathrm{i} + 1$ and $w = 2\mathrm{i} - 1$.

(b) $\dfrac{4}{9\sqrt{3}} - \dfrac{4}{9}\mathrm{i}$ or $\dfrac{4\sqrt{3}}{27} - \dfrac{4}{9}\mathrm{i}$

RVHS Prelim 9758/2017/01/Q6

Do not use a calculator in answering this question.

(a) Solve the simultaneous equations

$$z - 4w = 11 + 6\mathrm{i} \text{ and } 3z + 6\mathrm{i}w = 27$$

giving z and w in the form $x + \mathrm{i}y$ where x and y are real. [4]

(b) **(i)** The complex numbers z and w are given as $z = 4\left(\cos\dfrac{\pi}{3} - \mathrm{i}\sin\dfrac{\pi}{3}\right)$ and $w = 1 + \mathrm{i}\sqrt{3}$.

w^* denotes the conjugate of w. Find the modulus r and the argument θ of $\dfrac{w^*}{z^2}$, where $r > 0$ and $-\pi < \theta \le \pi$. [3]

(ii) Find the set of possible values of n such that $\left(\dfrac{w^*}{z^2}\right)^n$ is purely imaginary. [3]

Answers

(a) $w = -1 - i$, $z = 7 + 2\mathrm{i}$

(bi) $\dfrac{1}{8}$, $\dfrac{\pi}{3}$

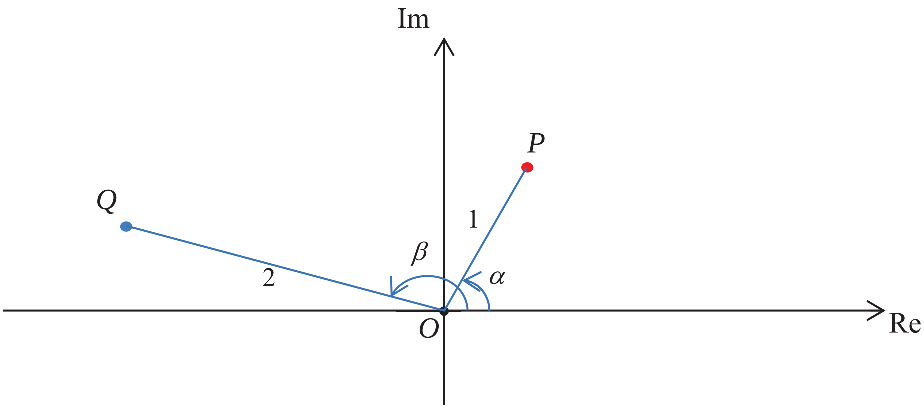
(bii) $\left\{n : n = \dfrac{3(2m+1)}{2},\ m \in \mathbb{Z}\right\}$

	SAJC Prelim 9758/2017/01/Q7
	(i) Show that for any complex number $z = r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$, and $-\pi < \theta \leq \pi$, $$\frac{z}{z-r} = \frac{1}{2} - \frac{1}{2}\left(\cot\frac{\theta}{2}\right)\mathrm{i}.$$ **[3]** **(ii)** Given that $z = 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{3}\right)}$ is a root of the equation $z^2 - 2z + 4 = 0$. State, in similar form, the other root of the equation. **[1]** **(iii)** Using parts **(i)** and **(ii)**, solve the equation $\frac{4w^2}{(w-1)^2} - \frac{4w}{w-1} + 4 = 0.$ **[4]**
	Answers ii) $z = 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$ iii) $w = \frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$ or $w = \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$

	SAJC Prelim 9758/2017/02/Q1
	Without the use of a calculator, find the complex numbers z and w which satisfy the simultaneous equations $$z - w\mathrm{i} = 3$$ $$z^2 - w + 6 + 3\mathrm{i} = 0$$ **[6]**
	Answers $z = 2\mathrm{i}$ or $z = -3\mathrm{i}$ $w = 2 + 3i$ or $w = -3 + 3\mathrm{i}$

	SRJC Prelim 9758/2017/01/Q1
	The complex numbers z and w satisfy the simultaneous equations $\mathrm{i}z + w = 2 + \mathrm{i}$ and $2w - (1+\mathrm{i})z = 8 + 4\mathrm{i}$.

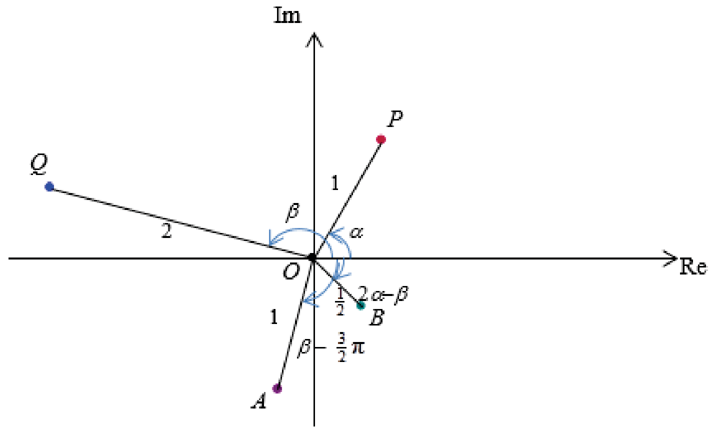
	Find z and w in the form of $a+\mathrm{i}b$, where a and b are real. [5]
	Answers $z=-1+\mathrm{i}$ and $w=3+2\mathrm{i}$

	SRJC Prelim 9758/2017/01/Q3
	For $\alpha, \beta \in \mathbb{R}$ such that $2\alpha < \beta$, the complex numbers $z_1 = \mathrm{e}^{\mathrm{i}\alpha}$ and $z_2 = 2\mathrm{e}^{\mathrm{i}\beta}$ are represented by the points P and Q respectively in the Argand diagram below.  Find the modulus and argument of the complex numbers given by $\frac{\mathrm{i}}{2}z_2$ and $\frac{{z_1}^2}{z_2}$. [4] Copy the given Argand diagram onto your answer script and indicate clearly the following points representing the corresponding complex numbers on your diagram. **(i)** A: $\frac{\mathrm{i}}{2}z_2$ [1] **(ii)** B: $\frac{{z_1}^2}{z_2}$ [1] You are expected to indicate clearly the relevant moduli and arguments for parts **(i)** and **(ii)** on your Argand diagram. If $\beta = \frac{11}{12}\pi$, find the smallest positive integer n such that the point representing the complex number $(z_2)^n$ lies on the negative real axis. [3]
	Answers

$\left|\frac{\mathrm{i}}{2}z_2\right| = 1,\ \arg(\frac{\mathrm{i}}{2}z_2) = \beta - \frac{3\pi}{2}$

$\left|\frac{z_1^{\ 2}}{z_2}\right| = \frac{1}{2},\ \arg\left(\frac{z_1^{\ 2}}{z_2}\right) = 2\alpha - \beta$

(i) & (ii)



Smallest n required = 12

TJC Prelim 9758/2017/01/Q8

(a) In an Argand diagram, points P and Q represent the complex numbers $z_1 = 2+3\mathrm{i}$ and $z_2 = \mathrm{i}z_1$.

(i) Find the area of the triangle OPQ, where O is the origin. [2]

(ii) z_1 and z_2 are roots of the equation $(z^2+az+b)(z^2+cz+d)=0$, where $a, b, c, d \in \mathbb{R}$. Find a, b, c and d. [4]

(b) Without using the graphing calculator, find in exact form, the modulus and argument of $v^* = \left(\frac{\sqrt{3}+\mathrm{i}}{-1+\mathrm{i}}\right)^{14}$. Hence express v in exponential form. [5]

Answers

(a)(i) $\frac{13}{2}$

(ii) $a=-4,\ b=13,\ c=6,\ d=13$

(b) $v = 2^7 \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$

TPJC Prelim 9758/2017/01/Q4

It is given that $z=-1-\mathrm{i}\sqrt{3}$.

(i) Given that $\dfrac{(\mathrm{i}z)^n}{z^2}$ is purely imaginary, find the smallest positive integer n. [4]

The complex number w is such that $|wz|=4$ and $\arg\left(\dfrac{w^*}{z^2}\right)=-\dfrac{5\pi}{6}$.

(ii) Find the value of $|w|$ and the exact value of $\arg(w)$ in terms of π. [3]

On an Argand diagram, points A and B represent the complex numbers w and z respectively.

(iii) Referred to the origin O, find the exact value of the angle OAB in terms of π. Hence, or otherwise, find the exact value of $\arg(z-w)$ in terms of π. [2]

Answers

(i) $\therefore$ smallest positive integer $n=5$.

(ii) $|w|=2$, $\arg(w)=\dfrac{13\pi}{6}$

(iii) **<u>Hence Method:</u>** $\arg(z-w)=-\left[\pi-\dfrac{\pi}{6}-\dfrac{\pi}{12}\right]$

$$=-\left[\frac{5\pi}{6}-\left(\frac{1}{2}\left\{\pi-\frac{5\pi}{6}\right\}\right)\right]$$

$$=-\frac{3\pi}{4} \quad \text{(exact)}$$

<u>Otherwise Method:</u>

$z-w=\left(-1-\sqrt{3}\right)+\left(-1-\sqrt{3}\right)\mathrm{i}$ $\qquad \arg(z-w)=-\left(\pi-\dfrac{\pi}{4}\right)=-\dfrac{3\pi}{4}$

TPJC Prelim 9758/2017/02/Q1

The cubic equation $az^3-31z^2+212z+b=0$, where a and b are real numbers, has a complex root $z=1-3\mathrm{i}$.

(i) Explain why the equation must have a real root. [2]

(ii) Find the values of a and b and the real root, showing your working clearly. [5]

Answers

(i)Since the **coefficients** of $az^3 - 31z^2 + 212z + b = 0$ are **all real**, **complex roots occur in conjugate pair**. Since a **cubic equation has three roots**, the third root must be a real root. (ii) $a = 25$, $b = 190$, $-\frac{19}{25}$

VJC Prelim 9758/2017/01/Q10
It is given that z_1, z_2 and z_3 are the roots of the equation $$2z^3 + pz^2 + qz - 4 = 0$$ such that $\arg z_1 < \arg z_2 < \arg z_3$ and $z_1 = 1 - \mathrm{i}\sqrt{3}$. Find the values of the real numbers p and q . [3] **(i)** Without using the calculator, find z_2 and z_3. [3] In an Argand diagram, points P, Q and R represent the complex numbers z_1, $w = \sqrt{2} + \mathrm{i}\sqrt{2}$ and $z_1 + w$ respectively and O is the origin. **(ii)** Express each of z_1 and w in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give r and θ in exact form. [2] **(iii)** Indicate P, Q and R on the Argand diagram and identify the type of the quadrilateral $OPRQ$. [3] **(iv)** Find the exact value of $\arg\left(z_1{}^4 w^*\right)$. [3]
Answers $p = -5, q = 10$ (i) $z_2 = \frac{1}{2}$, $z_3 = 1 + \sqrt{3}\mathrm{i}$ (ii) $z_1 = 2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}$, $w = 2\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$ (iii) rhombus (iv) $\frac{5\pi}{12}$

	YJC Prelim 9758/2017/01/Q5

	It is given that $z=\sqrt{3}+\mathrm{i}$ and $w=-1+\mathrm{i}$. **(i)** Without using a calculator, find an exact expression for $\dfrac{z^2}{w^*}$. Give your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [4] **(ii)** Find the exact value of the real number q such that $\arg\left(1-\dfrac{q}{z}\right)=\dfrac{\pi}{12}$. [3]
	Answers (i) $2^{\frac{3}{2}}\mathrm{e}^{-\mathrm{i}\frac{11\pi}{12}}$ (ii) $q=\sqrt{3}-1$

	YJC Prelim 9758/2017/02/Q3
	Do not use a calculator in answering this question. Given that $z=1+\mathrm{i}$ is a root of the equation $2z^4+az^3+7z^2+bz+2=0$, find the values of the real numbers a and b and the other roots. [5] Deduce the roots of the equation $2z^4+bz^3+7z^2+az+2=0$. [2]
	Answers $a=-5$, $b=-4$, $1-\mathrm{i}$, $\dfrac{1\pm\sqrt{7}\mathrm{i}}{4}$; $\dfrac{1-\mathrm{i}}{2}, \dfrac{1+\mathrm{i}}{2}, \dfrac{1-\sqrt{7}\mathrm{i}}{2}, \dfrac{1+\sqrt{7}\mathrm{i}}{2}$