Solution:

Using similar Δs, $\dfrac{y}{x+9} = \dfrac{4}{x}$

$$y = \frac{36+4x}{x}$$

$$y = \frac{36}{x} + 4$$

$$PA + AQ = x + 9 + y$$

$$= x + 9 + \frac{36}{x} + 4$$

$$= \left(x + \frac{36}{x} + 13\right) \text{ m}$$

Let $z = PA + AQ$

$$z = x + \frac{36}{x} + 13$$

$$\frac{dz}{dx} = 1 - \frac{36}{x^2}$$

For z to be minimum, $\dfrac{dz}{dx} = 0$

$$1 - \frac{36}{x^2} = 0$$

$$\frac{36}{x^2} = 1$$

$$x^2 = 36$$

$$x = 6 \quad \text{or} \quad x = -6 \text{ (N.A.)}$$

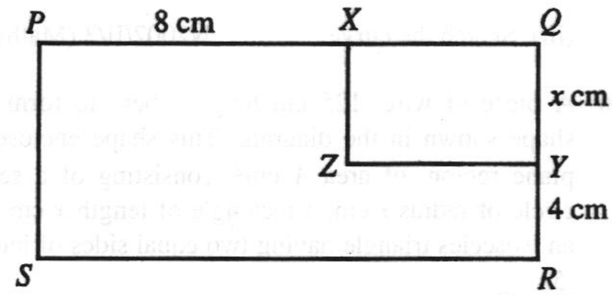
$$\frac{d^2z}{dr^2} = \frac{72}{x^3}$$

When $r = 6$, $\dfrac{d^2z}{dr^2} > 0$ (min)

∴ $PA + AQ$ is minimum when $x = 6$.

Past Years Examination Questions

1 The diagram shows the rectangles $PQRS$ and $XQYZ$, where $PX = 8$ cm, $QY = x$ cm, $YR = 4$ cm and the area of $XQYZ$ is 72 cm^2.



(i) Show that the area, A cm^2, of $PQRS$ is given by

$$A = 104 + 8x + \frac{288}{x}.$$

Given that x can vary,

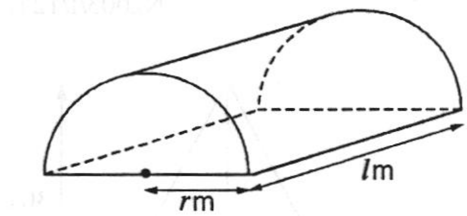
(ii) find an expression for $\dfrac{dA}{dx}$,

(iii) find the length and width of the rectangle $PQRS$ when its area is a minimum.

N2002/I/13*(b)* (AO Maths)

2 A curve has the equation $y = x^3 + 3x^2 - 2x + 12$. Find the x-coordinates of the turning points on the curve and determine the nature of these turning points. N2002/I/12*(a)* (AO Maths)

3



The diagram shows a greenhouse standing on a horizontal rectangular base. The vertical semicircular ends and the curved roof are made from polythene sheeting. The radius of each semicircle is r m and the length of the greenhouse is l m. Given that 120 m^2 of polythene sheeting is used for the greenhouse, express l in terms of r and show that the volume, V m^3, of the greenhouse is given by

$$V = 60r - \frac{\pi r^3}{2}. \quad [4]$$

Given that r can vary, find, to 2 decimal places, the value of r for which V has a stationary value. [3]

Find this value of V and determine whether it is a maximum or a minimum. [3]

N2002/II/12 *(EITHER)*

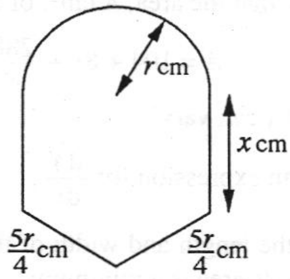
4 Find the coordinates of the stationary points of the curve $y = \dfrac{x}{x^2+1}$. N2002/II/3(*a*) (AO Maths)

5 (i) Find the coordinates of the stationary points on the curve $y = x^4 - 4x^3 + 27$.

(ii) Determine the nature of each of these stationary points.

(iii) Sketch the curve. N2002/II/3 (Maths C)

6 A piece of wire, 125 cm long, is bent to form the shape shown in the diagram. This shape encloses a plane region, of area A cm^2, consisting of a semi-circle of radius r cm, a rectangle of length x cm and an isosceles triangle having two equal sides of length $\dfrac{5r}{4}$ cm.



(i) Express x in terms of r and hence show that

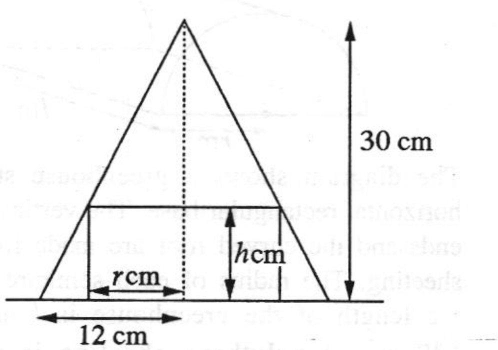
$$A = 125r - \frac{\pi r^2}{2} - \frac{7r^2}{4}.$$ [6]

Given that r can vary,

(ii) calculate, to 1 decimal place, the value of r for which A has a maximum value. [4]

N2003/I/12 (*EITHER*)

7



The diagram shows the cross-section of a hollow cone of height 30 cm and base radius 12 cm and a solid cylinder of radius r cm and height h cm. Both stand on a horizontal surface with the cylinder inside the cone. The upper circular edge of the cylinder is in contact with the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by

$$V = \pi(30r^2 - \tfrac{5}{2}r^3).$$ [4]

Given that r can vary,

(ii) find the volume of the largest cylinder which can stand inside the cone and show that, in this case, the cylinder occupies $\frac{4}{9}$ of the volume of the cone. [6]

[The volume, V, of a cone of height H and radius R is given by $V = \frac{1}{3}\pi R^2 H$.]

N2003/I/12 (*OR*)

8 A solid circular cylinder has height h cm and radius 4 cm. The total surface area of the cylinder is 600 cm^2. The volume of the cylinder is V cm^3.

(i) Express h in terms of r and hence show that $V = 300r - \pi r^3$.

(ii) Given that h and r can vary, find the maximum value of V.

N2003/I/16(*a*) (AO Maths)

9 Find the coordinates and the nature of each of the turning points on the curve $y = \dfrac{x^2}{x-1}$.

N2003/II/3(*a*) (AO Maths)

10 Each member of a set of curves has an equation of the form $y = ax + \dfrac{b}{x^2}$ where a and b are integers.

(i) For the curve where $a = 3$ and $b = 2$, find the area bounded by the curve, the x-axis and the lines $x = 2$ and $x = 4$. [4]

Another curve of this set has a stationary point at (2, 3).

(ii) Find the value of a and of b in this case and determine the nature of the stationary point. [6]

N2004/II/12 (*OR*)

11 Find the x-coordinates of all the stationary points on the curve $y = \dfrac{x^3}{(x+1)}$ stating, with reasons, the nature of each point. N2004/I/14 (Maths C)

12 A cylindrical metal tank, open at the top, has height h m and radius r m. The capacity of the tank is 3 m^3. The thickness of the metal may be neglected.

(i) Show that the exterior surface area, S m^2 of the metal used for the tank is given by $S = \pi r^2 + \dfrac{6}{r}$.

(ii) Given that r can vary, find the stationary value of S.

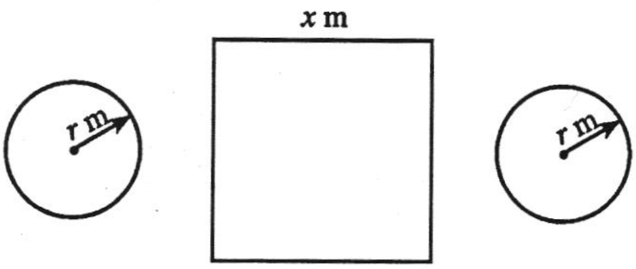
(iii) Determine the nature of this stationary value.

N2005/I/16 (*EITHER*) (AO Maths)

13 The curve $y = ax^2 + bx + 5$ has a stationary point at (2, 1). Find the value of each of the constants a and b.

N2007/I/6 (AO Maths)

14 A piece of wire of length 4 m is cut into three sections. Two of the sections are bent to form circles of radius r m and the other section is bent to form a square of side x m.



(i) Show that $x = 1 - \pi r$.

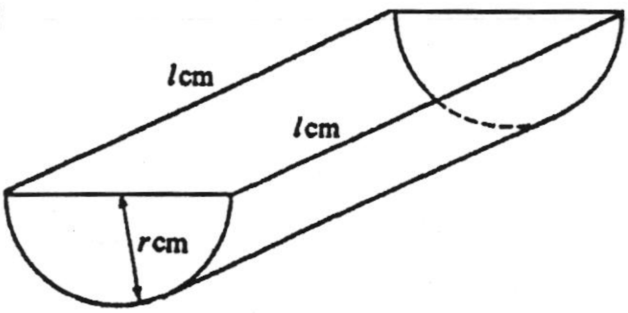
(ii) Show that the combined area, A m^2 of the three sections is given by $A = (\pi^2 + 2\pi)\, r^2 - 2\pi r + 1$.

(iii) Find the value of r for which A has a stationary value.

(iv) Determine, with explanation, whether the stationary value of A is a maximum or a minimum.

N2007/II/7 (AO Maths)

15



The diagram shows a tank, made of thin sheet metal, in the shape of half of a hollow cylinder. The length of the tank is l cm and each of the two semi-circular ends is of radius r cm. The total area of thin sheet metal is 8500 cm^2.

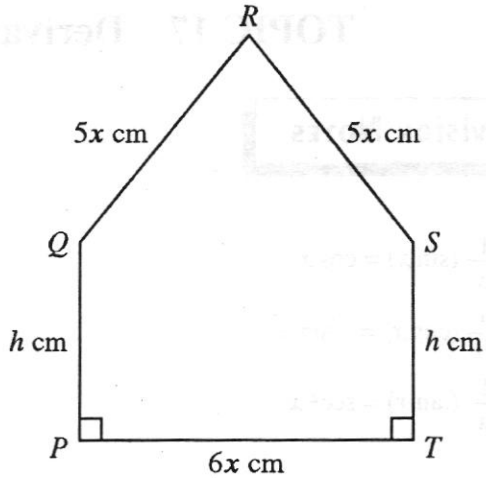
(i) Show that the volume, V cm^3, of the tank given by

$$V = 4250r = {}^{1}\!/_{2}\pi r^3. \qquad [4]$$

(ii) Given that r can vary, and that V has maximum value, find this maximum value of V. [4]

N2008/I/8 (Syll. 4018)

16 The diagram shows a glass window, $PQRST$, consisting of a rectangle $PQST$of height h cm and width $6x$ cm and an isosceles triangle QRS in which $QR = RS = 5x$ cm. The perimeter of the window is 360 cm.



(i) Show that the area of the window, A cm^2, is given by $A = 1080x - 36x^2$. [4]

Given that x can vary,

(ii) find the stationary value of A, [4]

(iii) determine whether this stationary value is a maximum or a minimum. [1]

N2008/I/13

17 Given that $y = \dfrac{\ln x}{x}$ for $x > 0$, find the set of values of x for which y is an increasing function of x. [5]

N2009/I/2