

1. [ACJC Prelims 17]

- (a) A biased cube with exactly one face painted red is thrown n times. Denoting the number of times the red face appears by X , it is found that $E(X) = \frac{40}{7}$ and $Var(X) = \frac{200}{49}$.

Find the value of n and hence find the probability that more than one quarter of the throws showed the red face. [5]

- (b) ** Each person in a large group of N people is asked to throw the same cube n times. Using a suitable approximation, determine the least value of N so that there is a probability of more than 0.9 that the mean number of red faces obtained per person is less than 6. [5]

2. [AJC Prelims 17]

A nursery sells a large number of rose seeds. 25% of the seeds are red rose seeds, and the rest are either yellow or pink rose seeds. The nursery sells the seeds in packs of 12, and each pack contains a random selection of rose seeds. For these packs, the mean number of yellow rose seeds is 3.6. A pack of rose seeds is chosen at random.

- (a) Show that the probability that the pack contains at most three yellow rose seeds is 0.4925. [2]

- (b) Find the probability that more than half of the seeds in the pack are either red or yellow rose seeds. [2]

A box contains 200 packs of seeds.

- (c) Find the probability that at least 30%, but less than 60% of the packs contain at most three yellow rose seeds. [2]

John buys a pack of rose seeds. His pack of seeds contains three red rose seeds, four yellow rose seeds and five pink rose seeds. His child randomly picks three seeds from the pack to plant them in a row. Find the probability that

- (d) there are at least two pink rose seeds planted, [3]

- (e) the third seed planted is a pink rose seed if it is known that at least two pink rose seeds are planted. [3]

3. [DHS Prelims 17]

Students pursuing a particular university course are required to take m modules in each semester. At the end of each module, the students have to take an examination which comprises n questions. It may be assumed that for each examination, the number of questions answered correctly by a randomly chosen student follows a binomial distribution $B(n, 0.6)$ with variance 24.

- (a) Verify that $n = 100$. [1]

To pass a module, a student must answer at least 50 questions correctly in the examination.

- (b) Find the most probable number of questions that a randomly chosen student answers correctly in an examination. [2]

- (c) Show that the probability that a randomly chosen student passes a module is 0.983. [1]

- (d) Given that a randomly chosen student is at most 90.4% confident of passing all his modules in a semester, find the least value of m . [3]

- (e) ** Forty students in this course are randomly selected and their marks for a particular examination are recorded. Use a suitable approximation to find the probability that on average at most 58 questions are answered correctly. [3]

4. [HCI Prelims 17]

The probability of a diner choosing a burger during his visit to Cheeky Chick Cafe is 0.05. Among the diners who visited Cheeky Chick Cafe, 20 diners are randomly chosen

- (a) Find the probability that more than 3 diners choose a burger. [2]

- (b) Find the smallest value of n such that there is more than 90% chance of less than n diners choosing a burger. [3]

- (c) The probability of a diner buying a drink in the cafe is p , where $p > 0.5$. Given that the variance of a diner buying a drink is 4.55, find the value of p . [2]

5. [IJC Prelims 17]

In a neighbourhood, it is known that 9% of the residents use the bicycle-sharing platform, ShareBike. A sample of n residents is selected at random and the number of residents who use ShareBike in the sample is denoted by the random variable X .

- (a) State, in context, an assumption needed for X to be well modelled by a binomial distribution. [1]
- (b) Explain why the assumption stated in part (a) may not hold in this context. [1]

Assume now that the assumption stated in part (a) does in fact hold.

- (c) Find the greatest value of n such that the probability that there is at least 1 resident using ShareBike is less than 0.99. [3]

It is now given that $n = 20$.

- (d) Find the probability that more than 2 but at most 5 residents use ShareBike. [2]
- (e) ** 40 such random samples are taken and the number of residents using ShareBike is being observed in each sample. Find the probability that the mean number of residents using ShareBike of these observations exceeds 2. [3]

Answers

1. (a) $n = 20, P(X > 5) = 0.528$.
(b) 83.
2. (a) 0.4925.
(b) 0.527.
(c) 0.999.
(d) $\frac{4}{11}$.
(e) $\frac{17}{24}$.
3. (b) 60.
(d) 6.
(e) 0.00491.
4. (a) 0.0159.
(b) $n = 3$.
(c) $p = 0.65$.
5. (a) The assumption is that the event of a resident using ShareBike or not is independent of any other residents in the neighbourhood.
(b) The assumption may not hold as usually families may use ShareBike together as they are going for the activity together.
(c) Greatest $n = 48$.
(d) 0.260.
(e) 0.161.