1. **[ACJC Prelims 17]**
   (a) A manager claims that a cup of coffee brewed by a particular barista contains at least 35 ml of coffee on average. A random sample of 80 cups of coffee brewed by the barista is examined and the quantity x ml of espresso coffee in each cup is measured. The results are summarized by $\sum(x-35) = -40$ and $\sum(x-35)^2 = 950$.
      i. Find unbiased estimates of the population mean and variance. [2]
      ii. Suggest a reason why, in this context, the given data is summarised in terms of $(x-35)$ rather than x. [1]
      iii. Test at the 10% significance level whether the managers claim is valid. [5]
   (b) A product designer claims that a new coffee machine brews coffee that contains 35 ml of coffee on average. The variance of the quantity of coffee in each cup is known to be 10.1 ml^2. A random sample of 80 cups of coffee made by the machine is measured. A test at the 10% significance level revealed that the product designers claim that each cup of coffee is 35 ml on average is valid.

      Find the range of values of the mean quantity of coffee in this sample, giving your answer correct to 3 decimal places. [4]

2. **[AJC Prelims 17]**
   A baker claims that the mean mass of his Xtra loaf of bread is 800 g. The mass of the loaves is known to have a standard deviation of 10.1 g. A random sample of 50 loaves was taken, and found to have a mean mass of 797.7 grams.
   (a) Test the bakers claim at the 5% level of significance. [4]
   (b) Meanwhile, a group of consumers used the same sample to carry out a different test. They conclude that the baker is overstating the mean mass at the k% significance level. Find the smallest value of k to three significant figures. [3]

   The bakery also claims that the average mass of a certain compound in each loaf of healthy bread is 150 mg. The mass of the compound in the loaves is normally distributed and the standard deviation is σ mg. A random sample of 60 loaves of healthy bread is taken, and the mass of compound in each loaf y mg is observed. The results are summarised as $\sum(y-150) = 60$.

   A test at 6% shows that the baker is understating the average mass of compound.
   (c) Find the possible values that σ can take. [4]

3. **[DHS Prelims 17]**
   (a) The centre thickness, X micrometres, of soft contact lenses from a certain company is a normally distributed random variable with mean μ. The company claims that the centre thickness of their lenses is at most 30 micrometres. A random sample of 60 contact lenses is measured. The results are summarised as follows.
   $$\sum(x-30) = 24 \qquad \sum(x-30)^2 = 144$$
      i. Test, at the 2.5% significance level, whether the claim is justified. [6]
      ii. Explain, in the context of the question, the meaning of "at the 2.5% significance level". [1]

(b) The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicles arrival is 7 minutes. A random sample of 30 passengers waiting times is obtained and the standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.

i. State appropriate hypotheses and the distribution of the test statistic used. [3]

ii. Find the range of values of the sample mean waiting time, $\bar{t}$. [3]

iii. A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]

4. **[HCI Prelims 17]**

The weight of a packet of Calhwa potato chips is known to have a mean of 84 grams and standard deviation 5 grams. The manufacturer claims that the average weight of a packet of potato chips is at least 84 grams. To test this claim, a random sample of 100 such packets of potato chips are selected and tested. The average weight of the 100 packets of potato chips in the sample is 82.9 grams.

(a) State appropriate hypotheses for the test, defining any symbols you use. [2]

(b) Test, at the 1% significance level, whether the manufacturer's claim is valid. [3]

(c) State what you understand by the expression 'at the 1% significance level' in part (b). [1]

(d) State, giving a reason, whether it is necessary for the weight of the packets of potato chips produced by a manufacturer to follow a normal distribution for the test in part (b) to be valid. [1]

(e) Another random sample of 100 packets of potato chips from another batch gives an average weight of t grams. Find the range of values of t such that there is enough evidence to conclude that the average weight of the packets of potato chips has changed at the 5% level of significance. [5]

Answers

1. (a) i. Unbiased estimate of population mean $= 34.5$.
   Unbiased estimate of population variance $= 11.8$.
   ii. Keeping the recorded values small since they are around 35 ml.
   iii. $p = 0.096213$. Reject H_0.
   (b) $34.416 \le \overline{x} \le 35.584$.

2. (a) $p = 0.107$. Do not reject H_0.
   (b) $k = 5.37$.
   (c) $0 \le \sigma \le 4.98$.

3. (a) i. $p = 0.0200$. Reject H_0.
   ii. It means that there is a probability of 0.025 of wrongly rejecting the claim that the mean centre thickness of the soft contact lenses is at most 30 micrometres.
   (b) i. $H_0 : \mu = 7, H_1 : \mu \neq 7$.
   $\overline{T} \sim N(7, \frac{4}{29})$ approximately by CLT since n is large.
   ii. $\overline{t} \le 6.04$ or $\overline{t} \ge 7.96$.
   iii. Reject H_0.

4. (a) Let X be the weight of a packet of Calhwa potato chips and μ denotes the population mean weight of a packet of potato chips in grams.
   $H_0 : \mu = 84, H_1 : \mu < 84$.
   (b) $p = 0.0139$. Do not reject H_0.
   (c) When the level of significance is set at 1%, there is 1% chance that we wrongly conclude the mean weight of a packet of potato chips is less than 84 grams when in fact the mean weight of a packet of potato chips is at least 84 grams.
   (d) Since the sample size $n = 100$ is sufficiently large, the sample mean weight of the packets of potato chips will be normally distributed by the Central Limit Theorem. Therefore it is not necessary to assume the weight of packets of potato chips follow a normal distribution.
   (e) $t \le 83.0$ or $t \ge 85.0$.