

# Random Variables Question Set

May 12, 2018

1. [JJC 15 Prelims (modified)]

A race requires each participant to complete a course involving cycling, swimming and running. Augustine intends to enter this race. His times, in minutes, that he takes to complete the three sections may be modeled as having independent normal distributions with means and standard deviations as shown in the table.

Section	Mean time	Standard deviation
Cycling	21.0	1.9
Swimming	13.5	1.5
Running	10.6	1.3

- (a) Find the probability that Augustine will complete the course in a total time of less than 47 minutes. [2]
- (b) In the actual race, Augustine and another participant, Anand, each complete the cycling and swimming sections in the same total time of 34.2 minutes.

Anand's times, in minutes, for completing the running section may be modeled as a normal distribution with mean 11.0 minutes and standard deviation 1.9 minutes.

- i. The record time taken to complete the course is 44.0 minutes. Determine, with justification, whether Augustine or Anand is more likely to break the record. [3]
- ii. Stating a necessary assumption, find the probability that the time taken by Augustine to complete the course will exceed four times the time taken by Anand on his running section. [4]

2. [JJC 15 Prelims (modified)]

Pauline frequently uses a social networking website to chat with her friends in the evening. On this site, she has a total of 120 friends. If she logs on to the site at 9.30 pm, on average, 35% of her friends will also be logged on to the site.

Pauline logs on to the site at 9.30 pm on a particular day. The number of her friends who are also logged on to the site is denoted by  $L$ .

- (a) State, in context, two assumptions needed for  $L$  to be well modeled by a binomial distribution. [2]
- (b) Explain why one of the assumptions stated may not be valid in this context. [1]

Assume now that these assumptions do in fact hold.

- (c) If 8 of her friends on this site are chosen at random, find the probability that more than half of them are logged on to the site. [2]
- (d) Find the greatest value of  $n$  such that the probability that at least  $n$  of her 120 friends are logged on to the site is more than 0.8. [3]
- (e) Find the mean and variance of  $L$ . [2]
- (f) Pauline wants to promote her blog shop on the website. Two price plans are offered: ‘Value’ and ‘Quality’.

Under the ‘Value’ price plan, no advertising fee is required if at most 40 of her 120 friends are logged on. If at least 41 friends and less than 50 friends are logged on, an advertising fee of \$40 is required. Otherwise an advertising fee of \$80 is required.

Under the ‘Quality’ price plan, there is a set up fee of \$10. Subsequently, a fee of 50 cents is charged for every friend that is logged in.

- i. Find the expected amount of advertising fee Pauline will pay under the ‘Value’ price plan. [3]
- ii. Should Pauline choose the ‘Value’ or ‘Quality’ price plan? Justify your reason with calculations. [3]
- iii. Find the variance of the amount of money Pauline pays under the ‘Quality’ price plan. [2]

3. [RVHS 15 Prelims(modified)]

The masses, in grams, of mobile phones sold by companies A and B have independent normal distributions with means and standard deviations as shown in the table.

	Mean weight	Standard deviation
Company A	130	6
Company B	$\mu$	$\sigma$

- (a) Find the probability that the total mass of 3 mobile phones from Company A is more than 400 g. [2]
- (b) The mass of a random mobile phone sold by Company B is equally likely to be less than 134 g and more than 146 g, with a probability of 0.234. Find  $\mu$  and  $\sigma$ . [3]
- (c) 10 mobile phones from Company A are taken.
  - i. Find the probability that the heaviest of them is at most 135 g. [2]
  - ii. Find the probability that more than 7 of them is less than 135 g. [2]
- (d) Let  $\mu = 136$  and  $\sigma = 8$ . Find the probability that the mass of a randomly chosen mobile phone sold by Company A is within 150 g of twice the mass of a randomly chosen mobile phone sold by Company B. [3]

4. [RVHS 15 Prelims (modified)]

A survey is administered to find out people's knowledge of Singapore's history. A person surveyed has to answer 8 multiple-choice questions, each with the same number of options given and has only 1 correct answer.

Suppose that for someone totally clueless of Singapore's history, there is a probability of 0.503 (correct to 3 significant figures) to get at most 1 correct answer when doing the survey, show that each multiple-choice question has 5 options. [2]

Assume that foreigners are totally clueless of Singapore's history. Find the most probable number of correct answers obtained by a foreigner who did the survey. [2]

A sample of 50 foreigners who did the survey is taken. Estimate the probability that the mean number of correct answers is at least 2. [3]

5. [SRJC 15 Prelims]

The mass of a randomly chosen turnip has mean 40 g and standard deviation of 3 g. If the probability that the mean mass of a large sample of  $n$  turnips is greater than 39.6 g exceeds 0.95, find the least value of  $n$ . [3]

6. [SRJC 15 Prelims (modified)]

In a factory manufacturing calculators, it is found that 1.5% of calculators produced are defective.

A box contains 90 calculators. Find the probability that

- (a) there are exactly 2 defective calculators in a box, [1]
- (b) the 90<sup>th</sup> calculator is the second defective calculator given that there are exactly 2 defective calculators in a box.
- (c) not more than 1 box of calculators, out of 60 boxes, contain more than 2 defective calculators. [3]

7. A discrete random variable  $X$  is defined by

$$P(X = x) = k(x - 1.5)^2 \quad \text{for } x = 0, 1 \text{ or } 2.$$

- (a) Find the value of  $k$ . [1]
- (b) Find  $E(X)$  and  $\text{Var}(X)$ . [3]
- (c) Write down the probability distribution function of  $Y = |X - 1|$  and hence find  $E(Y)$ . [2]
- (d) 50 observations of  $X$  was obtained. Find the probability that the mean of these 50 observations is within 0.1 of 0.5. [2]

## Answers

1. 0.755  
Augustine (0.269 vs 0.264)  
The time taken for Augustine to complete the course is independent from the time taken for Anand to complete the running section.  
0.554.
2. Friends are logged on to the site independently.  
The probability that any one friend is logged onto to the site is the same (0.35).  
The first assumption may not be valid as the log-ons of her friends may not be independent as her friends may be out together or arrange to chat on the site etc.  
The second assumption mentioned may not be valid as the probability of log on may vary from person to person because of different lifestyles and schedules etc.  
0.106  
38  
 $E(L) = 42, \text{Var}(L) = 27.3$ .  
27.5.  
Value (27.5 vs 31).  
6.83.
3. 0.168.  
140, 8.27.  
0.104.  
0.671.  
0.680.
4. 1.  
0.000544.
5. 153.
6. 0.238.  
0.0222.  
0.00289.
7.  $\frac{4}{11}$ .  
 $\frac{3}{11}, \frac{46}{121}$ .  
 $P(Y = 0) = \frac{1}{11}, P(Y = 1) = \frac{10}{11}, E(Y) = \frac{10}{11}$ .  
0.0721.