

Sampling Theory Discussion Problems

1. [ACJC 10 Prelims (modified)]

The marks in H2 mathematics for a certain examination board had a mean of 53.2 and a standard deviation of 11.1. A sample of 220 candidates of Tao Wan Junior College sat for this paper and obtained an average mark of 54.6.

Calculate the probability that the mean mark of this sample falls below 55. [2]

2. [CJC 10 Prelims]

Twelve-year old boys and girls have heights, in cm, that are assumed to be independent and normally distributed with means and standard deviations as shown in the following table.

	Mean Height	Standard Deviation
Boys	149	9
Girls	151	8

One boy and two girls are chosen at random.

- (a) Find the probability that the total height of the two girls is less than twice the height of the boy. [4]
- (b) Find the probability that only one of the children has height greater than 155 cm. [4]

A sample of n boys and a sample of 50 girls are chosen at random.

- (c) Find the least n such that the probability of the average height of the n boys exceeding the average height of the 50 girls is less than 0.1. [5]
- (d) Explain whether it is necessary to apply Central Limit Theorem for your calculation in part (c). [1]

3. [DHS 10 Prelims]

It is known that out of the two tasks, Task A and Task B, given in the Project Work question paper, 34% of Junior College (JC) students choose Task B.

- (a) The number of students who choose Task B in a class of 25 is denoted by Y . State the distribution of Y . [1]
- (b) Determine the mean and variance of Y . [2]
- (c) A random sample of 50 classes, each of size 25, is chosen from among the JCs. Estimate the probability that the mean number of students who choose Task B in these classes is at most 9. [2]

4. [HCI 10 Prelims]

The random variable X has mean, $\mu = 1$ and standard deviation, $\sigma = 0.8$. The sum of 60 independent observations of X is denoted by S .

State, with a reason, the distribution of S and find $P(55 < S < 65)$. [4]

5. [DHS 10 Prelims]

A school has 2428 students. To carry out data collection for his Project Work, Mylo obtains a list of all the students in school according to class and numbers them from 1 to 2428. He then selects a sample by the following method. Four fair dice are thrown. He adds up the scores on the four dice and selects the first student according to this number, and selects every 24th student on the list thereafter.

- (a) State the size of the sample. [1]
- (b) Explain briefly whether the following statements are true.
 - i. Each student in the school has an equal probability of being in the sample. [1]
 - ii. The students in the sample are selected independently of one another. [1]

6. [HCI 10 Prelims (modified)]

The amount of donation \$ x collected by each student at a particular college fund-raising campaign is recorded. The donation from a sample of 150 students is summarized by

$$\sum (x - 60) = 750 \quad \text{and} \quad \sum (x - 60)^2 = 5762.$$

- (a) Find the unbiased estimate of the population mean and show that the unbiased estimate of the population variance is 13.5, correct to 3 significant figures. [3]

It is now given that the mean amount collected by the college population is \$60.

- (b) State clearly the distribution of the mean amount collected by 150 students. [1]
- (c) Find the probability that the mean amount collected by 150 students is less than \$59.80. [1]
- (d) Explain whether it is necessary to apply Central Limit Theorem for your calculation in part (c). [1]
- (e) Find the smallest possible sample size if the probability that the mean amount collected exceeds \$60.50 is less than 0.04. [4]

7. [ACJC 10 Prelims]

The power consumption of a certain brand of light-bulb may be assumed to follow a normal distribution. Following a change in manufacturing process, a sample of 120 bulbs was tested. Denoting the power consumption in watts of a bulb by w , it was found that

$$\sum (w - 100) = 52 \quad \text{and} \quad \sum (w - 100)^2 = 223.$$

- (a) Find the unbiased estimates of the population mean and variance, correct to 3 decimal places. [3]
- (b) If ten light-bulbs are selected at random, estimate the probability that the total power consumption exceeds 1000 watts. [2]
- (c) * The population mean power consumption is denoted by μ watts. Using the sample data, a significance test of the null hypothesis $\mu = 100$ against the alternative hypothesis $\mu > 100$ is carried out at the $\alpha\%$ level of significance. Find the set of values of α for which the null hypothesis will be rejected. [4]

8. [AJC 10 Prelims]

A college has 1600 students. The college counselor wishes to estimate the average number of hours of sleep students have per week.

The number of hours of sleep per week is denoted by x hours. The sample values for a random sample of 40 students are summarized as follows:

$$\sum (x - 25) = 180.4, \quad \sum (x - 25)^2 = 2264.8.$$

- (c) Calculate unbiased estimates of the mean and variances of the number of hours of sleep per week by a student. [2]
- (d) Another random sample, of 60 students, is to be taken from the college. Estimate the probability that the mean value of x , for this sample, will be between 25 and 30. [2]
- (e) State two assumptions used in obtaining this estimate. [2]