1. [**TYS 2007 (modified)**]

   (a) Sketch, for the graphs of $y = \dfrac{20}{x+2}$ and $y = 10 - x^2$ on the same axes. Show clearly on your diagram the equations of the asymptotes of $y = \dfrac{20}{x+2}$. [3]

   (b) The graphs intersect on the y-axis. Find, correct to 3 decimal places, the x-coordinates of the point of intersection for which $x > 0$. [1]

   (c) Find $\displaystyle\int \dfrac{20}{x+2}\, \mathrm{d}x$ and $\int (10 - x^2)\, \mathrm{d}x$. [3]

   (d) Use your answers to parts (b) and (c) to find the area of the region, in the first quadrant, between the two graphs. [2]

2. [**TYS 2009**]

   (a) Sketch the graphs of $y = \sqrt{x}$ and $y = \frac{1}{2}x$ on a single diagram and write down the coordinates of the points where $y = \sqrt{x}$ and $y = \frac{1}{2}x$ intersect. [2]

   (b) Find $\int \sqrt{x}\, \mathrm{d}x$ and $\int \frac{1}{2}x\, \mathrm{d}x$. [2]

   (c) Without using a calculator, find the area of the region between the two graphs. [2]
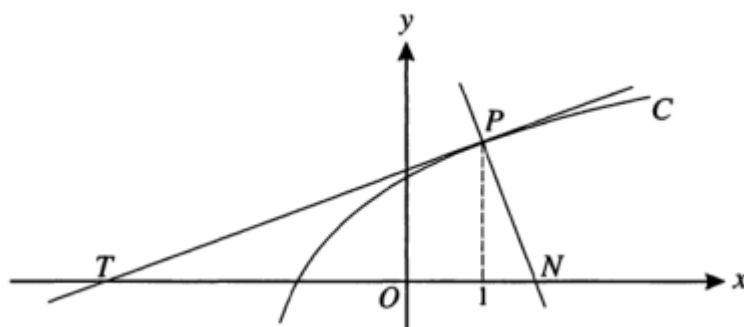
3. [**TYS 2008**]
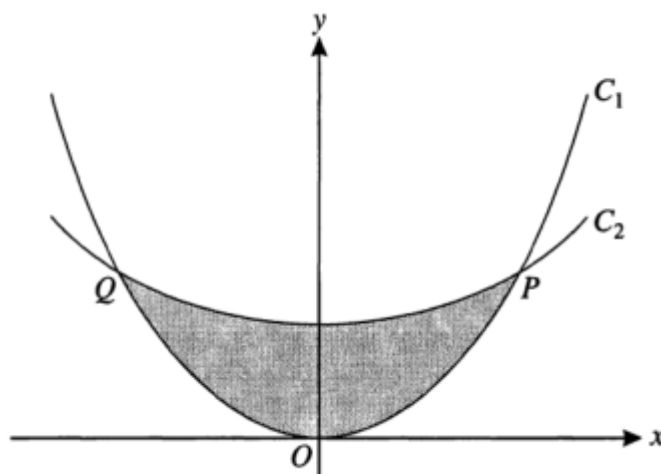
   The diagram shows the graphs of

   $$C_1 : y = 2x^2 \qquad \text{and} \qquad C_2 : y = x^2 + k^2,$$

   where k is a positive constant. The graphs intersect at P and Q, as shown.

   (a) Show that the x-coordinates of P and Q are k and $-k$ respectively. [1]

   (b) Find the exact value of the area of the shaded region between C_1 and C_2. [5]

4. **[TYS 2008 (modified)]**

The diagram shows the curve C with the equation $y = \ln(2x + 4)$. The point P on C has coordinates $(1, \ln 6)$.

(a) Find the coordinates of the intersection between C and the x-axis. [2]

(b) Show that the exact equation of T is $y = \dfrac{1}{3}x - \dfrac{1}{3} + \ln 6$. [3]

(c) ** Find the numerical value of the area bounded by C, the tangent to C at P and the line $x = -\dfrac{3}{2}$. Leave your answer correct to 4 decimal places. [3]

Answers

1. (a) $y = 0$ and $x = -2$.

   (b) 2.317.

   (c) $\displaystyle\int \frac{20}{x+2}\, \mathrm{d}x = 20 \ln |x+2| + C.$

   $\int (10 - x^2)\, \mathrm{d}x = 10x - \frac{x^3}{3} + C.$

   (d) 3.64 units^2.

2. (a) $(0,0)$ and $(4,2)$.

   (b) $\int \sqrt{x}\, \mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}} + C.$

   $\int \frac{1}{2}x\, \mathrm{d}x = \frac{x^2}{4} + C.$

   (c) $\frac{4}{3}$.

3. $\frac{4}{3}k^3$ units^2.

4. (a) $\left(-\frac{3}{2}, 0\right)$.

   (c) 0.5625 units^2.