

1. [TYS 2010]

For events A and B it is given that $P(A) = 0.7$, $P(B) = 0.6$ and $P(A|B') = 0.8$. Find

- (a) $P(A \cap B')$, [2]
- (b) $P(A \cup B)$, [2]
- (c) $P(B' \cap A)$. [2]

For a third event C , it is given that $P(C) = 0.5$ and that A and C are independent.

- (d) Find $P(A' \cap C)$. [2]
- (e) Hence state an inequality satisfied by $P(A' \cap B \cap C)$. [1]

For (e), try both the 1 mark version (original question) and the 4 mark version (find the 'best possible' inequality for $P(A' \cap B \cap C)$).

2. [TYS 2015]

For events A, B and C it is given that $P(A) = 0.45$, $P(B) = 0.4$, $P(C) = 0.3$ and $P(A \cap B \cap C) = 0.1$. It is also given that events A and B are independent, and that events A and C are independent.

- (a) Find $P(B|A)$. [1]
- (b) Given also that events B and C are independent, find $P(A' \cap B' \cap C')$. [3]
- (c) Given instead that events B and C are not independent, find the greatest and least possible values of $P(A' \cap B' \cap C')$. [4]

3. [TYS 2018]

The events A, B and C are such that $P(A) = a$, $P(B) = b$ and $P(C) = c$. A and B are independent events. A and C are mutually exclusive events.

- (a) Find an expression for $P(A' \cap B')$ and hence prove that A' and B' are independent events. [2]
- (b) Find an expression for $P(A' \cap C)$. Draw a Venn diagram to illustrate the case when A' and C' are also mutually exclusive events. (You should not show event B on your diagram.) [2]

You are now given that A' and C' are not mutually exclusive, $P(A) = \frac{2}{5}$, $P(B \cap C) = \frac{1}{5}$ and $P(A' \cap B' \cap C') = \frac{1}{10}$.

- (c) Find exactly the maximum and minimum possible values of $P(A \cap B)$.

4. [TYS 2011 (modified)]

A committee of 3 people is chosen at random from a group consisting of 6 women and 4 men. The number of women on the committee is denoted by R .

- (a) Find the probability that $R = 3$. [3]
- (b) Find the probability distribution of R and hence find $E(X)$ and $Var(X)$. [4]
- (c) The most probable number of women on the committee is denoted by r . By using the fact that $P(R = r) > P(R = r + 1)$, show that r satisfies the inequality

$$(r + 1)!(5 - r)!(2 - r)!(r + 2)! > r!(6 - r)!(3 - r)!(r + 1)!$$

- and use this inequality to find the value of r . [5]