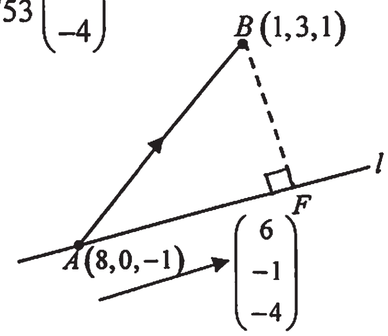
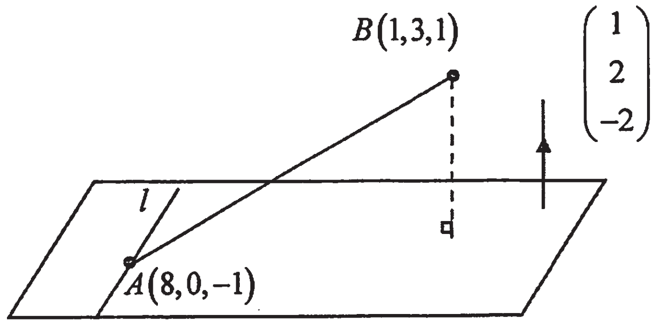
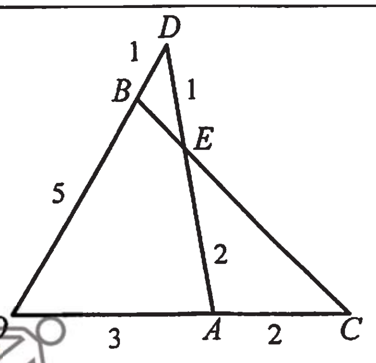
Qn	Solution	Comments
1i	$f(2)=f(1)=1$ $f(5)=\left[f(2)\right]^2-2=-1$ $f(10)=f(5)=-1$	Read qn carefully, f is defined over positive integers. $2n+1$ means odd integers, $2n$ means even integers. We should not be finding terms like $f\left(\frac{1}{2}\right)$.
ii	Since f(10) = f(5), f is not a one-one function $\therefore f^{-1}$ does not exist.	Need to mention the 2 values of n that have the same image under f. Usually, a graphical explanation is accompanied by a graph.
2i	Equation of l is $\underset{\sim}{r}=\begin{pmatrix}8\\0\\-1\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}, s\in\mathbb{R}$	
	Method 1: $\overrightarrow{OA}=\begin{pmatrix}8\\0\\-1\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}1\\3\\1\end{pmatrix}, \therefore \overrightarrow{AB}=\begin{pmatrix}-7\\3\\2\end{pmatrix}$ $\overrightarrow{AF}=\left(\begin{pmatrix}-7\\3\\2\end{pmatrix}\cdot\frac{1}{\sqrt{53}}\begin{pmatrix}6\\-1\\-4\end{pmatrix}\right)\frac{1}{\sqrt{53}}\begin{pmatrix}6\\-1\\-4\end{pmatrix}$ $=\frac{-53}{53}\begin{pmatrix}6\\-1\\-4\end{pmatrix}$ $=\begin{pmatrix}-6\\1\\4\end{pmatrix}$  $\overrightarrow{OF}=\overrightarrow{OA}+\overrightarrow{AF}=\begin{pmatrix}8\\0\\-1\end{pmatrix}+\begin{pmatrix}-6\\1\\4\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}$	When using the projection method, be mindful of the direction of the projection vector, i.e. if we are using $\overrightarrow{AB}$, then $\overrightarrow{AF}$ is the projection vector. If we are using $\overrightarrow{BA}$, then $\overrightarrow{FA}$ is the projection vector. The formula for projection vector has no modulus notation.
	Method 2: $\overrightarrow{OF}=\begin{pmatrix}8\\0\\-1\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}$, for some s $\overrightarrow{BF}=\begin{pmatrix}8\\0\\-1\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}-\begin{pmatrix}1\\3\\1\end{pmatrix}=\begin{pmatrix}7\\-3\\-2\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}$	

Qn	Solution	Comments
	$\left(\begin{pmatrix}7\\-3\\-2\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}\right)\cdot\begin{pmatrix}6\\-1\\-4\end{pmatrix}=0$ $53+53s=0\Rightarrow s=-1$ $\overrightarrow{OF}=\begin{pmatrix}8\\0\\-1\end{pmatrix}-\begin{pmatrix}6\\-1\\-4\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}$	
ii	**Method 1:** $\begin{pmatrix}8\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix}=8-2=6\ \therefore A(8,0,-1)$ lies in p. $\begin{pmatrix}6\\-1\\-4\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix}=6+2-8=0\ .\ l\perp\begin{pmatrix}1\\-2\\2\end{pmatrix}\ \therefore l$ is parallel to p. Since l is parallel to p, and A lies in p, l lies in p.	This is a show question Need to show all points circled.
	Method 2: $\left(\begin{pmatrix}8\\0\\-1\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}\right)\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix}=(8-2)+s(6+2-8)$ $=6+0s$ $=6$ Since $\underset{\sim}{r}=\begin{pmatrix}8\\0\\-1\end{pmatrix}+s\begin{pmatrix}6\\-1\\-4\end{pmatrix}$ satisfies the equation of p for all values of s, l lies in p.	This is a show question Need to show all points circled.
iii	Shortest distance = length of projection of $\overrightarrow{AB}$ on $\begin{pmatrix}1\\-2\\2\end{pmatrix}$ $=\left\lvert\begin{pmatrix}-7\\3\\2\end{pmatrix}\cdot\frac{1}{3}\begin{pmatrix}1\\-2\\2\end{pmatrix}\right\rvert=\frac{9}{3}=3$	The distance required is parallel to the normal vector, hence we find the length of projection onto the normal vector.

Qn	Solution	Comments
		Note: For this question, it happens that the point F is also the foot of perpendicular of B to p. In general, it may not be true. See diagram
iv	$\left\|\overrightarrow{BA}\times \underset{\sim}{n}\right\|$ is the length of projection of $\overrightarrow{BA}$ onto p.	Similar to the note above, it happens that AF is the answer to this question. In general, it may not be true.
3i	$c=1$. For g to be a function, every element in D_g needs to have an image under g. 1 has no image under g. Hence, it has to be removed from D_g for g to be a function.	This question requires the definition of function.
Ii	Let $y=\dfrac{2x+5}{1-x}$ $y-xy=2x+5$ $x(2+y)=y-5$ $x=\dfrac{y-5}{2+y}$ $g^{-1}(x)=\dfrac{x-5}{2+x}$ $D_{g^{-1}}=R_g=\mathbb{R}\setminus\{-2\}\quad \left(\text{or } (-\infty,-2)\cup(-2,\infty)\right)$	Find R_g by sketching the graph of $y=g(x)$ and finding the range of g. There are certain notations that are acceptable, **do not** mix and match to give wrong notations like $(-\infty,\infty), x\neq -2$. $D_{g^{-1}}=x\in\mathbb{R}, x\neq -2$ is also poor notation.
iii	$D_g=(-\infty,1)\cup(1,\infty)$, $R_g=(-\infty,-2)\cup(-2,\infty)$ Since $R_g \not\subseteq D_g$, g^2 does not exist.	Need to show both D_g and R_g in your working.
4	$\overrightarrow{OC}=\frac{5}{3}\underset{\sim}{a}$, $\overrightarrow{OD}=\frac{6}{5}\underset{\sim}{b}$  $\overrightarrow{OE}=\dfrac{\overrightarrow{OA}+2\overrightarrow{OD}}{3}$ $=\dfrac{\underset{\sim}{a}+\frac{12}{5}\underset{\sim}{b}}{3}$ $=\frac{1}{3}\underset{\sim}{a}+\frac{4}{5}\underset{\sim}{b}$ $\overrightarrow{BC}=\frac{5}{3}\underset{\sim}{a}-\underset{\sim}{b}$ $\overrightarrow{BE}=\frac{1}{3}\underset{\sim}{a}+\frac{4}{5}\underset{\sim}{b}-\underset{\sim}{b}=\frac{1}{3}\underset{\sim}{a}-\frac{1}{5}\underset{\sim}{b}$	"OA produced" means extended in the direction of OA. To show collinearity, need to show two of the three vectors $\overrightarrow{BC},\overrightarrow{CE},\overrightarrow{BE}$ are parallel, **and** that the vectors have a common point. We do not divide by vectors.

Qn	Solution	Comments
	Since $\overrightarrow{BC}=5\overrightarrow{BE}$, BC is parallel to BE and B is a common point, C, E and B are collinear.	
	$\overrightarrow{AC}=\frac{2}{3}\underset{\sim}{a},\ \overrightarrow{AE}=\frac{1}{3}\underset{\sim}{a}+\frac{4}{5}\underset{\sim}{b}-\underset{\sim}{a}=-\frac{2}{3}\underset{\sim}{a}+\frac{4}{5}\underset{\sim}{b}$ Area of $\Delta ACE=\frac{1}{2}\left\|\overrightarrow{AC}\times\overrightarrow{AE}\right\|=\frac{1}{2}\left\|\frac{2}{3}\underset{\sim}{a}\times\left(-\frac{2}{3}\underset{\sim}{a}+\frac{4}{5}\underset{\sim}{b}\right)\right\|$ $=\frac{1}{45}\left\|12\underset{\sim}{a}\times\underset{\sim}{b}-10\underset{\sim}{a}\times\underset{\sim}{a}\right\|=\frac{4}{15}\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|\quad[\because \underset{\sim}{a}\times\underset{\sim}{a}=\underset{\sim}{0}]$ $=\frac{4}{15}\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\sin 60°=\frac{2\sqrt{3}}{15}\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|$	This is a show question, hence we will need to explain what happened to $\underset{\sim}{a}\times\underset{\sim}{a}$. $\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|=\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\sin\theta$. This is given in the notes.
5i	$u_r-u_{r+1}=\frac{1}{\sqrt{r}}-\frac{1}{\sqrt{r+1}}$ $=\frac{\sqrt{r+1}-\sqrt{r}}{\sqrt{r}\sqrt{r+1}}\times\frac{\sqrt{r+1}+\sqrt{r}}{\sqrt{r+1}+\sqrt{r}}$ $=\frac{r+1-r}{(r+1)\sqrt{r}+r\sqrt{r+1}}$ $=\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}$	After combining into a single fraction, we need to get $(r+1)\sqrt{r}+r\sqrt{r+1}$ in the denominator. $(r+1)\sqrt{r}+r\sqrt{r+1}$ $=\sqrt{r}\sqrt{r+1}\left(\sqrt{r+1}+\sqrt{r}\right)$ Hence, we need to multiply numerator and denominator by $\sqrt{r+1}+\sqrt{r}$.
ii	$\frac{1}{4\sqrt{3}+3\sqrt{4}}+\frac{1}{5\sqrt{4}+4\sqrt{5}}+\ldots+\frac{1}{(N+1)\sqrt{N}+N\sqrt{N+1}}$ $=\sum_{r=3}^{N}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}$ $=\sum_{r=3}^{N}(u_r-u_{r+1})$ $\begin{array}{rlcl} = & u_3 & - & u_4 \\ + & u_4 & - & u_5 \\ + & u_5 & - & u_6 \\ & & \vdots & \\ + & u_{N-2} & - & u_{N-1} \\ + & u_{N-1} & - & u_N \\ + & u_N & - & u_{N+1} \end{array}$ $=u_3-u_{N+1}$ $=\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{N+1}}$	

Qn	Solution	Comments
iia	$\frac{1}{26\sqrt{25}+25\sqrt{26}}+\frac{1}{27\sqrt{26}+26\sqrt{27}}+\ldots+\frac{1}{49\sqrt{48}+48\sqrt{49}}$ $=\sum_{r=25}^{48}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}$ $=\sum_{r=3}^{48}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}-\sum_{r=3}^{24}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}$ $=\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{49}}-\left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{25}}\right)$ $=\frac{2}{35}$	These 2 questions require us to use the answer in part **(ii)**, hence clear working on how we used the answer in part **(ii)**
iib	Replace r with $r+1$ in $\sum_{r=3}^{N-1}\frac{1}{r\sqrt{r-1}+(r-1)\sqrt{r}}$ $\sum_{r=3}^{N-1}\frac{1}{r\sqrt{r-1}+(r-1)\sqrt{r}}$ $=\sum_{r+1=3}^{r+1=N-1}\frac{1}{(r+1)\sqrt{(r+1)-1}+(r+1-1)\sqrt{r+1}}$ $=\sum_{r=2}^{N-2}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}$ $=\sum_{r=3}^{N-2}\frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}\left(\text{or }+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)$ $=\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{N-1}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}\left(\text{or }\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{N-1}}\right)$	
6i	Equation of p_1 is $\underset{\sim}{r}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=0$ and equation on line AB is $\underset{\sim}{r}=\begin{pmatrix}3\\1\\2\end{pmatrix}+s\begin{pmatrix}1\\-2\\2\end{pmatrix}, s\in\mathbb{R}$ $\left(\begin{pmatrix}3\\1\\2\end{pmatrix}+s\begin{pmatrix}1\\-2\\2\end{pmatrix}\right)\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=0$ $2+2s=0\Rightarrow s=-1$ $\overrightarrow{OB}=\begin{pmatrix}3\\1\\2\end{pmatrix}-\begin{pmatrix}1\\-2\\2\end{pmatrix}=\begin{pmatrix}2\\3\\0\end{pmatrix}$ The coordinates of B is (2, 3, 0).	Read question carefully. Answers to be given as coordinates, not vectors.

Qn	Solution	Comments
ii	$\cos\left(\cos^{-1}\left(\frac{1}{9}\right)\right)=\frac{\begin{pmatrix}-4\\a\\1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{4^2+a^2+1}\sqrt{1}}$ $(\because$ Scalar product is positive$)$ $\frac{1}{9}=\frac{1}{\sqrt{17+a^2}}$ $17+a^2=81$ $a=8$ or $-8(\text{NA } \because a>0)$	For this question, we have to perform dot product with $\begin{pmatrix}0\\0\\1\end{pmatrix}$ because we need both sides of the equation to be positive.
iii	Method 1: $\overrightarrow{BC}=18\left(\frac{1}{\sqrt{16+64+1}}\right)\begin{pmatrix}-4\\8\\1\end{pmatrix}=\begin{pmatrix}-8\\16\\2\end{pmatrix}$ $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}=\begin{pmatrix}2\\3\\0\end{pmatrix}+\begin{pmatrix}-8\\16\\2\end{pmatrix}=\begin{pmatrix}-6\\19\\2\end{pmatrix}$ Method 2: $\overrightarrow{BC}=k\begin{pmatrix}-4\\8\\1\end{pmatrix}$, $k>0$ because $\overrightarrow{BC}$ is in the direction of $\begin{pmatrix}-4\\8\\1\end{pmatrix}$ $BC=\left\|k\begin{pmatrix}-4\\8\\1\end{pmatrix}\right\|=9k=18\Rightarrow k=2$ $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}=\begin{pmatrix}2\\3\\0\end{pmatrix}+2\begin{pmatrix}-4\\8\\1\end{pmatrix}=\begin{pmatrix}-6\\19\\2\end{pmatrix}$	Read question carefully: "in the direction of" does not mean "parallel". It has to be a positive factor of $\begin{pmatrix}-4\\8\\1\end{pmatrix}$. Furthermore, this is a show question. If there are 2 possible values of k, we need to reject with reason.
Iv	Method 1: $\overrightarrow{CD}=\begin{pmatrix}0\\7\\-4\end{pmatrix}-\begin{pmatrix}-6\\19\\2\end{pmatrix}=\begin{pmatrix}6\\-12\\-6\end{pmatrix}=6\begin{pmatrix}1\\-2\\-1\end{pmatrix}$ A normal vector to the plane $=\begin{pmatrix}1\\-2\\-1\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}1\\-2\\5\end{pmatrix}$	For this question, only 1 particular vector $\underset{\sim}{n}$ is accepted as the answer, because p is specified to be 34. Contrast with a question that asks for an equation of a plane in the form $\underset{\sim}{r}\cdot\underset{\sim}{n}=p$.

Qn	Solution	Comments
	$\underset{\sim}{n} = k\begin{pmatrix}1\\-2\\5\end{pmatrix}$ $\begin{pmatrix}0\\7\\-4\end{pmatrix}\cdot k\begin{pmatrix}1\\-2\\5\end{pmatrix} = -34k$ $-34k = 34 \Rightarrow k = -1$ $\underset{\sim}{n} = -\begin{pmatrix}1\\-2\\5\end{pmatrix} = \begin{pmatrix}-1\\2\\-5\end{pmatrix}$ Or $\underset{\sim}{r}\cdot\begin{pmatrix}1\\-2\\5\end{pmatrix} = \begin{pmatrix}0\\7\\-4\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\5\end{pmatrix} = -34$ $\underset{\sim}{r}\cdot\begin{pmatrix}1\\-2\\5\end{pmatrix} = -34$ -------- (1) $(1)\times(-1):\ \underset{\sim}{r}\cdot\begin{pmatrix}-1\\2\\-5\end{pmatrix} = 34$ Method 2: Let $\underset{\sim}{n} = \begin{pmatrix}x\\y\\z\end{pmatrix}$. $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix} = 0 \Rightarrow 2x + y = 0$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}0\\7\\-4\end{pmatrix} = 34 \Rightarrow 7y - 4z = 34$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}-6\\19\\2\end{pmatrix} = 34 \Rightarrow -6x + 19y + 2z = 34$ By GC, $x = -1, y = 2, z = -5$ $\therefore \underset{\sim}{n} = \begin{pmatrix}-1\\2\\-5\end{pmatrix}$	Hence, after finding $\underset{\sim}{n}$ from cross product, we need to check if the scalar product with position vector do indeed get 34.
7i	$f^2(40) = 0.9(0.9(40) + 4.5) + 4.5$ $= 0.9^2(40) + (0.9)4.5 + 4.5$ $f^3(40) = 0.9(0.9^2(40) + (0.9)4.5 + 4.5) + 4.5$ $= 0.9^3(40) + (0.9)^2 4.5 + (0.9)4.5 + 4.5$	From 0.9^n, we can guess that this is a GP question with common ratio 0.9. Hence, when simplifying $f^2(40)$ and $f^3(40)$, we need to keep 0.9 as part of the expression. We should avoid just pressing calculator and losing ability

Qn	Solution	Comments
	$\mathrm{f}^n(40)=0.9^n(40)+(0.9)^{n-1}4.5+(0.9)^{n-2}4.5+\ldots+4.5$ $=0.9^n(40)+\dfrac{4.5(1-0.9^n)}{1-0.9}$ $=0.9^n(40)+45(1-0.9^n)$ $=45+0.9^n(40-45)$ $=45-0.9^n(5)$	to recognize number pattern.
ii	$\sum_{n=j+1}^{2j}(\mathrm{f}^n(40)+4n+1)$ $=\sum_{n=j+1}^{2j}(45-0.9^n(5))+\sum_{n=j+1}^{2j}(4n+1)$ $=45j-\dfrac{0.9^{j+1}(5)(1-0.9^j)}{1-0.9}+\dfrac{j}{2}(4j+5+8j+1)$ $=45j-0.9^{j+1}(50)(1-0.9^j)+j(6j+3)$ Since j is large, $0.9^j\approx 0$ $\sum_{n=j+1}^{2j}(\mathrm{f}^n(a)+4n+1)\approx 45j+6j^2+3j=6j^2+48j$	
iii	$\sum_{n=1}^{\infty}(\mathrm{f}^n(40)-45)=\sum_{n=1}^{\infty}(0.9^n(-5))$ $=\dfrac{0.9(-5)}{1-0.9}$ $=-45$	
8ai	$\mathrm{P}(M'\cap W)=\dfrac{108+68}{500}=\dfrac{44}{125}(\text{or } 0.352)$	$\mathrm{P}(M'\cap W)\neq\mathrm{P}(M')\times\mathrm{P}(W)$ If LHS = RHS, the events M' and W need to be independent.
ii	$\mathrm{P}(A\|W)=\dfrac{\mathrm{n}(A\cap W)}{\mathrm{n}(W)}$ $=\dfrac{108}{108+59+68}$ $=\dfrac{108}{235}(\approx 0.460)$ It is the probability that a complaint of a product under warranty is regarding appearance.	
iii	Since $\mathrm{P}(A)=\dfrac{108+46}{500}=\dfrac{77}{250}(=0.308)\neq\mathrm{P}(A\|W)$, A and W are not independent. OR	

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td></td><td>$\mathrm{P}(A)=\frac{108+46}{500}=\frac{77}{250}$, $\mathrm{P}(W)=\frac{108+59+68}{500}=\frac{47}{100}$,
$\mathrm{P}(A\cap W)=\frac{108}{500}=\frac{27}{125}$
$\mathrm{P}(A)\mathrm{P}(W)=0.14476\neq \mathrm{P}(A\cap W)$
Hence, A and W are not independent.</td><td></td></tr>
<tr><td>b</td><td>Let X be the number of complaints that are regarding a product under warranty, out of 10.
$X\sim B(10,0.47)$
$\mathrm{P}(X\leq n)<0.3$
By GC,
<table><tr><th>n</th><th>$\mathrm{P}(X\leq n)$</th></tr><tr><td>2</td><td>0.07915</td></tr><tr><td>3</td><td>0.2255</td></tr><tr><td>4</td><td>0.45263</td></tr></table>
Largest value of $n=3$.</td><td>"at most n" means less than or equal to n.</td></tr>
<tr><td>9i</td><td>$\mathrm{P}(A'\cap B')=1-\frac{5}{9}=\frac{4}{9}$</td><td></td></tr>
<tr><td>(ii)</td><td>$\mathrm{P}(A|B)=\mathrm{P}(B|A)$
$\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}$
$\mathrm{P}(A)=\mathrm{P}(B)$

$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
$\frac{5}{9}=\mathrm{P}(A)+\mathrm{P}(A)-\mathrm{P}(A)\mathrm{P}(A)$
$9[\mathrm{P}(A)]^2-18\mathrm{P}(A)+5=0$
$\mathrm{P}(A)=\frac{1}{3}$ or $\frac{5}{3}$ (Reject $\because 0\leq \mathrm{P}(A)\leq 1$)

$\mathrm{P}(A\cap B)=\left(\frac{1}{3}\right)^2=\frac{1}{9}$</td><td></td></tr>
<tr><td>10i</td><td>$\mathrm{P}(W=1)=\mathrm{P}(MMW)+\mathrm{P}(MWM)+\mathrm{P}(WMM)$
$=\frac{10-n}{10}\times\frac{n}{9}\times\frac{n-1}{8}\times 3$
$=\frac{n(n-1)(10-n)}{240}$

Alternative Method:</td><td>This is not a binomial distribution. The probability of choosing a woman to be in the committee is not constant across the 3 selections.</td></tr>
</table>

Qn	Solution	Comments
ii	$$\mathrm{P}(W=1)=\frac{{}^{n}C_2\times{}^{10-n}C_1}{{}^{10}C_3}=\frac{\frac{n(n-1)}{2!}\times\frac{10-n}{1!}}{120}$$ $$=\frac{n(n-1)(10-n)}{240}$$	

w	$\mathrm{P}(W=w)$
0	$\frac{n(n-1)(n-2)}{720}$ $\left(\text{or } \frac{{}^{n}C_3}{120}\right)$
1	$\frac{n(n-1)(10-n)}{240}$
2	$\frac{n(10-n)(9-n)}{240}$ $\left(\text{or } \frac{n\left({}^{10-n}C_2\right)}{120}\right)$
3	$\frac{(10-n)(9-n)(8-n)}{720}$ $\left(\text{or } \frac{{}^{10-n}C_3}{120}\right)$

ii

$$\mathrm{E}(W)=\frac{n(n-1)(10-n)}{240}+2\left(\frac{n(10-n)(9-n)}{240}\right)+3\left(\frac{(10-n)(9-n)(8-n)}{720}\right)$$

$$2.1=\frac{10-n}{720}\left(3n(n-1)+6n(9-n)+3(9-n)(8-n)\right)$$

$$=\frac{10-n}{240}\left(n(n-1)+2n(9-n)+(9-n)(8-n)\right)$$

$$=\frac{10-n}{240}\left(n^2-n+18n-2n^2+72-17n+n^2\right)$$

$$=0.3(10-n)$$

$0.3n=0.9$

$n=3$

Or By GC, $n=3$

iii

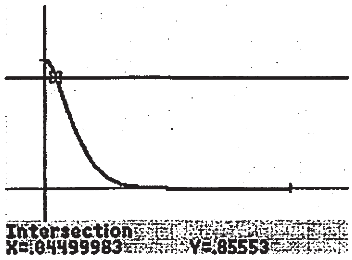
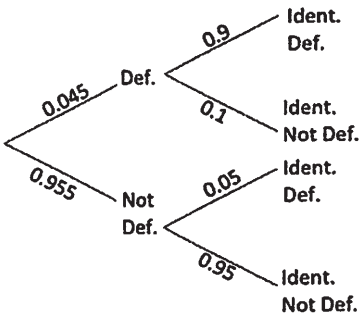
w	0	1	2	3
$\mathrm{P}(W=w)$	$\frac{1}{120}$	$\frac{7}{40}$	$\frac{21}{40}$	$\frac{7}{24}$

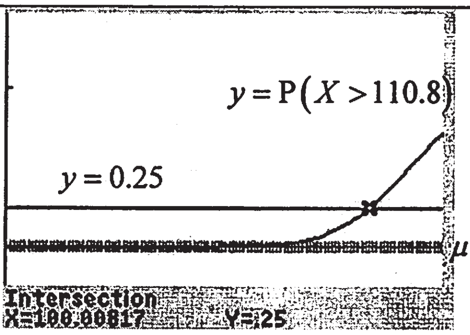
By GC, $\mathrm{Var}(W)=0.49$

11i

1. The probability of a defective sneaker remains constant throughout the sample of 15.
2. Defective sneakers occur independently of each other.

Comments: There is no such thing as independent probabilities. It is incorrect to say "defective sneakers are independent of each other".

Qn	Solution	Comments
ii	Let X be the number of defective sneakers, out of a sample of 15. $X \sim \mathrm{B}(15, p)$ $X + X' = 15 \Rightarrow X' = 15 - X$ $\mathrm{P}(X' \geq 14) = \mathrm{P}(15 - X \geq 14)$ $0.85553 = \mathrm{P}(X \leq 1)$ Using GC, $p = 0.045000$  $\mathrm{P}(X \leq 2) = 0.97239$ $\approx 0.972 (3\text{dp})$	Read question carefully, "at least 14 sneakers are <u>non-defective</u>." You will need to define your variables clearly. For show questions, we show the rounding off to the required degree of accuracy.
iii	Let Y be the number of defective sneakers, out of a sample of 5. $X \sim \mathrm{B}(5, 0.045000)$ $\mathrm{P}(\text{Satisfactory}) = \mathrm{P}(X \leq 2) + \mathrm{P}(X = 3)\mathrm{P}(Y = 0)$ $= 0.991$	
iv	 $\mathrm{P}(\text{identified not defective}) = 0.0450 \times 0.1 + 0.955 \times 0.95$ $= 0.912$	
12i	$\mathrm{P}(Y < 2) = 0.894$	
ii	$\mathrm{P}(X > 110.8) = 0.25$ Method 1: $\mathrm{P}\left(Z > \frac{110.8 - \mu}{16}\right) = 0.25$ $\frac{110.8 - \mu}{16} = 0.67449$ $\mu = 100.008$ $\approx 100.0 (1\text{dp})$ Method 2: By GC, Plot $Y_1 = $ normalcdf(110.8,$_E$99.X,16) $Y_2 = 0.25$	For show questions, we show the rounding off to the required degree of accuracy.

Qn	Solution	Comments
	 $\mu = 100.008$ $\approx 100.0 (1\text{dp})$	
iii	$\mathrm{P}(X_1 + X_2 > 3X_3) = \mathrm{P}(X_1 + X_2 - 3X_3 > 0)$ $\mathrm{E}(X_1 + X_2 - 3X_3) = 100.0 + 100.0 - 300.0 = -100.0$ $\mathrm{Var}(X_1 + X_2 - 3X_3) = 16^2 + 16^2 + (-3)^2 16^2 = 2816$ $\therefore X_1 + X_2 - 3X_3 \sim \mathrm{N}(-100.0, 2816)$ $\mathrm{P}(X_1 + X_2 - 3X_3 > 0) = 0.0298 \quad (\text{or } 0.0297)$ The number of minutes of telephone calls made during a one-month period is independent of that during other one-month periods.	The first month is independent of the second month, so we do not say $2X > 3X$ Assumptions should be given in the context of the question.
Iv	Let $\$T$ be the cost of Mark's mobile phone bill $T = 0.05X + 0.02(1000)Y = 0.05X + 20Y$ $\mathrm{E}(T) = 0.05(100.0) + 20(1) = 25$ $\mathrm{Var}(T) = 0.05^2(16^2) + 20^2(0.8^2) = 256.64$ $\therefore T \sim \mathrm{N}(25, 256.64)$ $\mathrm{P}(T > 30) = 0.377$	Note the difference in units between megabytes and gigabytes
v	The answer in part (iv) is greater because the event "cost of Marcus' telephone calls exceeds \$5 and the cost of his data use exceeds \$25" is a subset of the event "total cost of Marcus's mobile phone bill exceeds \$30".	