VICTORIA JUNIOR COLLEGE

JC 2 COMMON TEST 2 2018

H2 Mathematics **9758/01**

Paper 1 **3 hours**

Additional Materials: Answer Paper
Graph Paper (Upon Request)
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages

[Turn over

Section A: Pure Mathematics [59 Marks]

1 The function f is defined for positive integers n as follows:

$$f(1)=1,\quad f(2n)=f(n) \quad \text{and} \quad f(2n+1)=\left[f(n)\right]^2-2.$$

(i) Find f(2), f(5) and f(10). [2]

(ii) State whether the inverse of f exists, justifying your answer. [1]

2 The line l has equation $\dfrac{x-8}{6}=\dfrac{y}{-1}=\dfrac{z+1}{-4}$ and $A(8,0,-1)$ is a point on l. The point B has coordinates $(1,3,1)$.

(i) Find the position vector of the point F on l which is closest to B. [3]

(ii) Explain why l lies in the plane p with equation $x-2y+2z=6$. [2]

(iii) Find the shortest distance from B to p. [2]

(iv) Let $\mathbf{n}$ denote a unit vector perpendicular to plane ~~Π~~ p. Give the geometrical meaning of $\left|\overrightarrow{BA}\times\mathbf{n}\right|$. [1]

3 The functions g is defined by

$$g : x \mapsto \frac{2x+5}{1-x},\quad x\in\mathbb{R},\quad x\neq c.$$

(i) State the value of c and explain why this value has to be excluded from the domain of g. [2]

(ii) Find $g^{-1}(x)$ and state the domain of g^{-1}. [3]

(iii) Determine if composite function g^2 exist. [2]

4 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively.

The point C on OA produced is such that $OA:AC=3:2$, and the point D on OB produced is such that $OB:BD=5:1$. The point E on AD is such that $AE:ED=2:1$. Show that C, E and B are collinear. [4]

It is given that the angle AOB is $60°$. Use vector product to show that the area of triangle ACE can be written as $k\,|\mathbf{a}||\mathbf{b}|$, where k is a constant to be determined exactly. [4]

5 **(i)** Show that $u_r - u_{r+1} = \frac{1}{(r+1)\sqrt{r} + r\sqrt{r+1}}$, where $u_r = \frac{1}{\sqrt{r}}$. [2]

(ii) Hence, find $\frac{1}{4\sqrt{3}+3\sqrt{4}} + \frac{1}{5\sqrt{4}+4\sqrt{5}} + \ldots + \frac{1}{(N+1)\sqrt{N}+N\sqrt{N+1}}$ in terms of N. [3]

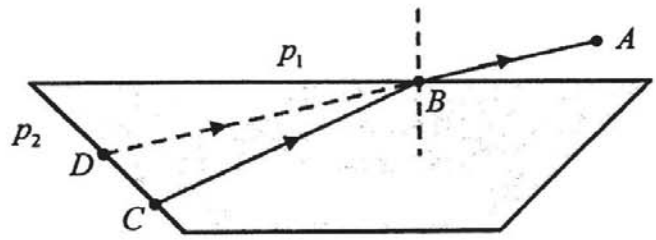
(iii) Using your answer in **(ii)**,

(a) find the exact value of

$$\frac{1}{26\sqrt{25}+25\sqrt{26}} + \frac{1}{27\sqrt{26}+26\sqrt{27}} + \ldots + \frac{1}{49\sqrt{48}+48\sqrt{49}}.$$ [2]

(b) express $\sum_{r=3}^{N-1} \frac{1}{r\sqrt{r-1}+(r-1)\sqrt{r}}$ in terms of N. [3]

6



The diagram above shows the vertical cross-section of a slab of glass in the form of a trapezoidal prism, where the top surface is a plane p_1 and the left side of the glass is a plane p_2. C and D are points in p_2. The light from a particle placed at C travels in a straight line to B in the glass. The light is refracted at B and travels in a straight line to $A(3,1,2)$ in the air. To an observer at A, the particle at C appears to be at $D(0,7,-4)$.

The plane p_1 has equation $z = 0$ and AB is parallel to $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

(i) Find the coordinates of B. [3]

$\overrightarrow{BC}$ is in the direction of $-4\mathbf{i} + a\mathbf{j} + \mathbf{k}$, where a is a positive constant. The line BC makes an angle of $\cos^{-1}\left(\frac{1}{9}\right)$ with the normal to p_1 at B.

(ii) Find the value of a. [3]

(iii) Given that the distance BC is 18, show that the position vector of C is $-6\mathbf{i} + 19\mathbf{j} + 2\mathbf{k}$. [2]

The plane p_2 has equation $\mathbf{r} \cdot \mathbf{n} = 34$.

(iv) Given that p_2 is perpendicular to a plane with equation $2x + y = 7$, find $\mathbf{n}$. [4]

7 The function f is defined for all real values of x by

$$f(x) = 0.9x + 4.5 \ .$$

(i) By considering $f^2(40)$ and $f^3(40)$, show that $f^n(40) = 45 - 0.9^n(5)$. [4]

(ii) Show that $\sum_{n=j+1}^{2j} \left(f^n(40) + 4n + 1\right) \approx 6j^2 + 48j$, given that j is a very large positive integer. [5]

(iii) Evaluate $\sum_{n=1}^{\infty} \left(f^n(40) - 45\right)$. [2]

[Note: f^2 denotes ff.]

Section B: Probability and Statistics [41 Marks]

8 A manufacturer wishes to find out the reasons for consumer complaints for his product. He conducted an analysis of 500 consumer complaints. The table below shows the results of the analysis in which the reason for complaints and the period of the complaint was lodged were recorded.

Reason for complaints	Appearance	Mechanical	Electrical
During warranty period	108	59	68
After warranty period	46	109	110

(a) One complaint is selected at random.

A, M and E are the events that the complaint selected is regarding appearance, mechanical and electrical respectively.

W is the event that the complaint selected occurs during warranty period.

(i) Calculate $P(M' \cap W)$. [1]

(ii) Calculate $P(A|W)$ and explain the meaning of this probability in the context of the question. [3]

(iii) Determine whether A and W are independent. Justify your conclusion. [2]

(b) Ten complaints are selected at random, one by one and with replacement. Find the largest value of n such that the probability that at most n complaints are regarding a product under warranty is less than 0.3. [2]

9 Two independent events A and B are such that $P(A|B) = P(B|A)$ and $P(A \cup B) = \frac{5}{9}$, find

(i) $P(A' \cap B')$, [1]

(ii) $P(A \cap B)$. [4]

10 A board of directors consists n men and $(10-n)$ women, where $3 \leq n \leq 7$. A committee consisting 3 randomly chosen people is to be formed. Let W be the number of women in the committee.

(i) Show that $P(W=1) = \frac{n(n-1)(10-n)}{240}$ and find, in terms of n, the probability distribution of W. [4]

(ii) Given $E(W) = 2.1$, find n. [2]

(iii) Hence, find Var(W). [1]

11 A shoe company manufactures a large number of sneakers every day. A small proportion, p, of these sneakers is defective. A check is carried out each day by taking a random sample of 15 sneakers and examining them for defects.

(i) State, in context, two assumptions needed for the number of defective sneakers in the sample to be well modelled by a binomial distribution. [2]

For the rest of this question, assume that the binomial law holds.

(ii) The probability that at least 14 of the sneakers in the sample are non-defective is 0.85553. Show that the probability that at most 2 sneakers in the sample are defective is 0.972, correct to 3 decimal places. [3]

If exactly 3 sneakers are defective, a further sample of 5 sneakers is taken. A day's production is accepted as satisfactory in either one of the following cases:

- The number of defective sneakers in the sample of 15 is at most 2;
- The number of defective sneakers in the sample of 15 is 3, **and** there is no defective sneaker in the sample of 5.

(iii) Find the probability that the day's production is accepted as satisfactory. [2]

Subsequently, the manufacturer realises a fault in the check for defective sneakers. If a sneaker is defective, there is a 90% chance that the check correctly identifies it as defective. If the sneaker is not defective, there is a 5% chance that the check incorrectly identifies it as defective.

(iv) Find the probability that a sneaker identified as not defective. [2]

[Turn Over

12 In this question you should state clearly the values of the parameters of any normal distribution you use.

Over a one-month period, Mark makes X minutes of telephone calls and uses Y gigabytes of data on his mobile phone. X and Y are independent random variables with the distribution $X \sim \mathrm{N}\left(\mu, 16^2\right)$ and $Y \sim \mathrm{N}\left(1, 0.8^2\right)$ respectively.

(i) Find the probability that over a one-month period, Mark uses less than 2 gigabytes of data. [1]

(ii) There is a 25% chance that Mark makes more than 110.8 minutes of telephone call in a month. Show that $\mu = 100.0$, correct to 1 decimal place. [2]

(iii) Find the probability that, over three one-month periods, the total number of minutes of telephone calls made by Mark in the first two months is more than thrice the number of minutes made by him in the third month. [3]

State an assumption needed for the calculation above to be valid. [1]

Mark opted for a Pay As You Use plan. For this payment plan, there is no monthly subscription. Mobile phone's calls cost \$0.05 per minute and data costs \$0.02 per megabyte.

[One gigabyte is equal to 1000 megabytes]

(iv) Find the probability that, over a one-month period, the total cost of Mark's mobile phone bill exceeds \$30. [3]

(v) Explain, without evaluating, whether your answer in **(iv)** is greater than or less than the probability that, over a one-month period, the cost of Mark's telephone calls exceeds \$5 and the cost of his data use exceeds \$25. [2]

[End of Paper]