1. [IJC/2013/Promo/6]

Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P on AB is such that $AP : PB = 2 : 3$. It is given that $|\mathbf{a}| = \sqrt{5}$, $|\mathbf{b}| = 3$ and OP is perpendicular to AB.

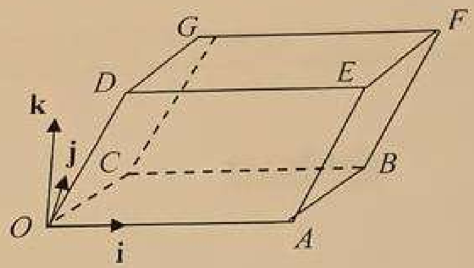
(i) Show that $\mathbf{a} \cdot \mathbf{b} = -3$. [3]

(ii) Find the size of angle AOB. [2]

(iii) Find the length of projection of $\overrightarrow{OB}$ onto OA. [1]

2. [HCI/2013/Promo/12 (modified)]

An art structure, which is a parallelpiped (made of 6 faces of parallelograms) has a horizontal base $OABC$, with OA, OC and OD as its three sides and remaining vertices are B, E, F, and G as shown in the diagram below.



It is given that $\overrightarrow{OA} = 5\mathbf{i}$ and $\overrightarrow{OC} = \mathbf{i} + 7\mathbf{j}$. The lines l_1 and l_2 have equations given by

$l_1 : \mathbf{r} = (5+\lambda)\mathbf{i} + (7\lambda - 14)\mathbf{j} + 6\mathbf{k}$, where λ is a real parameter and

$l_2 : 3x = z + 15$, $y = 0$.

E and F are on line l_1 and A and E are on line l_2.

(i) Show that $\overrightarrow{OE} = 7\mathbf{i} + 6\mathbf{k}$. [2]

(ii) Find the equation, in scalar product form, of the plane $ABFE$. [3]

(iii) Find the length of projection of $\overrightarrow{AE}$ onto the base $OABC$. Hence, or otherwise, find the area of the projection of the plane $ABFE$ onto the base. [3]

(iv) Find the equation of the line l_3, which is the reflection of line AE about the base $OABC$. [2]

(v) An architect wants to add a shelter which has the plane equation $x + ay + bz = c$, where a, b and c are unknown constants. He wants the shelter to meet the plane $ABFE$ at EF. What can be said about the values of a, b and c? [2]

3. [IJC/2013/Promo/ 11]

(i) Find a vector equation of the line through the points A and B with position vectors $3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$ and $-\mathbf{i}+12\mathbf{j}+9\mathbf{k}$ respectively. [2]

(ii) The perpendicular to this line from the point C with position vector $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ meets the line at the point N. Find the position vector of N. [3]

(iii) Find a Cartesian equation of the line AC. [2]

(iv) Use a vector product to find the exact area of triangle OAB. [3]

4. [AJC/2012/I/10 (modified)]

The two planes p_1 and p_2, given by the equations $2x + y = 1$ and $8x + ay + z = 4$ respectively, meet at a line L which contains the point $A(0,1,0)$.

(i) Show that $a = 4$ and hence find the vector equation of the line L. [3]

(ii) Another point B lies in the plane p_1 such that AB is perpendicular to the line L. Show that

$\overrightarrow{AB}$ is parallel to $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

If the distance of B from p_2 is 5 units, find the possible position vectors of point B. [5]

(iii) Find the acute angle between line AB and p_2. [2]

5. Points A, B and C are such that $\overrightarrow{OA} = i + j + k$, $\overrightarrow{OB} = -3i + 2j - 5k$ and $\overrightarrow{OC} = i + 6j - k$. M is on BC and the ratio of $BM : MC$ is λ: 1.

(i) Find an expression for $\overrightarrow{OM}$ in terms of λ. [1]

(ii) Calculate the angle AOB. [2]

(iii) Find the shortest distance from B to $\overrightarrow{OA}$ produced in exact form. [2]

(iv) Show that the line passing through B and C is parallel to $\overrightarrow{OA}$. [2]

(v) Hence, find the area of ΔAOM in exact form, stating your reasons clearly. [2]

6. [SRJC/2013/JC1 Midyear/8]

The line l_1 has equation $x+1=y=2-z$. The line l_2 passes through the point $A(3,0,-6)$ and is parallel to the vector $\mathbf{i}-2\mathbf{k}$.

(i) Given that the point $B(-1,0,2)$ lies on l_1, show that it also lies on l_2. [1]

(ii) Given that the acute angle between the lines l_1 and l_2 is denoted by θ, find $\cos\theta$ in exact form. Hence find the shortest distance from A to the line l_1 in exact form. [5]

(iii) Find the position vector of F, the foot of the perpendicular from the point $C(-2,1,5)$ to l_2. Hence find the coordinates of the reflection of C in the line l_2. [6]

7. [NYJC/2013/JC1 Mid-year/8]

The points A, B and C have coordinates $(1,-1,3)$, $(2,-2,1)$ and $(10,-5,1)$ respectively.

(i) Show that a vector equation of the line l passing through B and C can be expressed as $\mathbf{r}=\begin{pmatrix}2\\-2\\1\end{pmatrix}+\lambda\begin{pmatrix}8\\-3\\0\end{pmatrix}, \lambda\in\mathbb{R}$. [1]

(ii) Determine the possible coordinates of the point D on l such that $CD=2\sqrt{73}$ units. [4]

The plane Π_1 passes through the point A and contains l. Show that the equation of Π_1 can be written in the form $\mathbf{r}\cdot\begin{pmatrix}6\\16\\a\end{pmatrix}=b$ where the values of a and b are to be determined. [3]

The plane Π_2 has equation $\mathbf{r}\cdot\begin{pmatrix}-6\\-16\\\alpha\end{pmatrix}=\beta$. Determine the values of α and β such that Π_1 and Π_2 are parallel, at a distance $\frac{30}{\sqrt{317}}$ units apart and on the opposite sides of the origin. [5]

8. The line l_1 has equation $\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+s\begin{pmatrix}1\\0\\-1\end{pmatrix}, s\in\mathbb{R}$, and the plane p_1 has equation $7x-12y+z=-16$.

(i) Find the acute angle between l_1 and p_1. [2]

(ii) Show that the plane p_2, which is perpendicular to p_1 and contains l_1, has the equation $\mathbf{r}\cdot(3\mathbf{i}+2\mathbf{j}+3\mathbf{k})=17$. [4]

(iii) The planes p_1 and p_2 intersect in a line, l_2. Find the vector equation of l_2. [2]

(iv) Given a point A with coordinates $(-7,13,-5)$, find the position vector of the foot of perpendicular from A to p_1. Hence, or otherwise, find the coordinates of the point B, which is the mirror image of A in p_1. [5]