MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 1

MATHEMATICS

8863/01

Paper 1

October/November 2007

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

Singapore Examinations and Assessment Board

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

Section A: Pure Mathematics [40 marks]

1 **(i)** Find the numerical value of the derivative of 3^x when $x = 2$. [1]

(ii) Hence find the equation of the tangent to the graph of $y = 3^x$ at the point where $x = 2$, giving your answer in the form $y = mx + c$. [2]

2 The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 1, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \mathrm{e}^x, \quad x \in \mathbb{R}.$$

(i) Write down $\mathrm{fg}(x)$. [1]

(ii) Sketch the graph of $y = \mathrm{fg}(x)$, showing clearly any asymptotes. State the exact coordinates of any points where the graph crosses the axes. [3]

(iii) Write down $\mathrm{gf}(x)$. [1]

(iv) Find $(\mathrm{gf})^{-1}(x)$. [2]

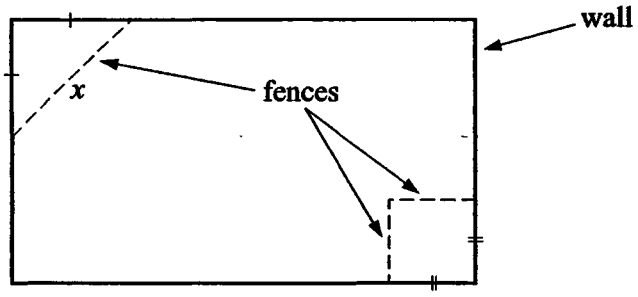
3 **(i)** Sketch, for $x \geqslant 0$, the graphs of $y = \dfrac{20}{x+2}$ and $y = 10 - x^2$ on the same axes. [2]

(ii) The graphs intersect on the y-axis. Find, correct to 3 decimal places, the x-coordinate of the point of intersection for which $x > 0$. [1]

(iii) Find $\displaystyle\int \frac{20}{x+2}\,\mathrm{d}x$ and $\displaystyle\int (10 - x^2)\,\mathrm{d}x$. [3]

(iv) Use your answers to parts **(ii)** and **(iii)** to find the area of the region, in the first quadrant, between the two graphs. [2]

4



The diagram shows a large rectangular field surrounded by a wall. The broken lines represent fences. The corner shapes are an isosceles triangle and a square. The length of the fence bordering the triangle is x metres.

(i) Explain why the area of the triangle is $\frac{1}{4}x^2$ m^2. [2]

The total length of the fences is 100 m. The total area of the triangle and the square is A m^2.

(ii) Show that $A = 2500 - 50x + \frac{1}{2}x^2$. [3]

(iii) Use differentiation to find the value of x for which A is a minimum. State the corresponding minimum value of A and explain briefly how you can tell that it is a minimum rather than a maximum. [4]

(iv) Find the largest value that A can take, given that $10 \leqslant x \leqslant 80$. Show clearly how you obtain your answer. [2]

5 Without using a calculator, solve the simultaneous equations

$$y = 2x^2 + 3x + 2 \quad \text{and} \quad y = 2x + 3. \qquad [3]$$

Hence solve the inequality

$$2x^2 + 3x + 2 \geqslant 2x + 3. \qquad [2]$$

Hence, using a sketch of the graph of $x = \cos\theta$, solve the inequality

$$2\cos^2\theta + 3\cos\theta + 2 \geqslant 2\cos\theta + 3$$

for $0° \leqslant \theta \leqslant 540°$. [6]

Section B: Statistics [60 marks]

6 A manufacturer produces packets of margarine. The mass of margarine in a packet has a normal distribution with mean 502 g and standard deviation 0.8 g. Find the proportion of packets which contain less than 500 g of margarine. [2]

The manufacturer increases the mean amount of margarine in a packet to μ g. The standard deviation remains unchanged. Only 1 packet in 1000, on average, now contains less than 500 g. Find μ, correct to 1 decimal place. [3]

7 A school has a canteen where students can buy their lunch. Each day most, but not all, students buy their lunch in the canteen.

The headteacher wants to find out what students think of the lunches provided in the canteen. On one particular day she selects a sample of students to interview from those buying their lunch by

- choosing at random one of the first 10 students to buy their lunch,
- then choosing every 10th student after the first student chosen.

(i) What is this type of sampling method called? [1]

(ii) State one advantage and one disadvantage of the sampling method used in this context. [2]

(iii) Describe an alternative sampling method which would be better in this case. [2]

8 Seven cities in a certain country are linked by rail to the capital city. The table below shows the distance of each city from the capital and the rail fare from the city to the capital.

City	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Distance, x km	124	44	76	148	16	180	104
Rail fare, \$$y$	156	53	99	169	23	177	138

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Calculate the product moment correlation coefficient. [1]

You are given that the regression line of y on x has equation $y = 16.7 + 1.01x$, where the coefficients are given correct to 3 significant figures.

(iii) Calculate the equation of the regression line of x on y, giving your answer in the form $x = a + by$. [1]

(iv) Use the appropriate regression line to estimate

(a) the rail fare from a city that is 28 km from the capital, [2]

(b) the distance of a city from the capital if the rail fare is \$198. [2]

(v) Comment briefly on the reliability of the estimates in part **(iv)**. [2]

9 A random variable X has a binomial distribution with $n = 6$ and probability of success p. Write down an expression, in terms of p, for $P(X = 4)$. [1]

It is given that $p = \frac{1}{4}$.

(i) Find $P(X = 4)$, giving your answer as a fraction. [1]

(ii) The mean and standard deviation of X are denoted by μ and σ respectively. Find $P(\mu - \sigma < X < \mu + \sigma)$, correct to 2 decimal places. [5]

10 Bottles of a particular brand of washing-up liquid are said to contain 500 ml. A random sample of 50 bottles is taken and the volumes of liquid in the bottles are measured. The volumes, x ml, are summarised by $\Sigma(x - 500) = -35.8$ and $\Sigma(x - 500)^2 = 150.5$.

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Assuming a normal distribution, test at the 5% significance level whether the population mean volume is less than 500 ml. [4]

(iii) State, giving a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

11 The table below shows the results of a survey of the 120 cars in a car park, in which the colour of each car and the gender of the driver were recorded.

	Male	Female
Green	18	12
Blue	48	22
Red	6	14

One of the cars is selected at random.

M is the event that the car selected has a male owner.
G is the event that the car selected is green.
B is the event that the car selected is blue.
R is the event that the car selected is red.

(i) Find the following probabilities:

(a) $\mathrm{P}(M)$, [1]

(b) $\mathrm{P}(M \cap G)$, [1]

(c) $\mathrm{P}(M \cup B)$, [1]

(d) $\mathrm{P}(M \mid R')$. [1]

(ii) Determine whether the events M and G are independent, justifying your answer. [2]

(iii) It is given that bicycle racks are fitted to 20% of the green cars, 30% of the blue cars and 5% of the red cars. One of the cars is selected at random and found to have a bicycle rack fitted. What is the probability that it is a blue car? [5]

[Turn over

12 Men and women have masses, in kg, that are normally distributed with means and standard deviations as shown in the following table.

	Mean mass	Standard deviation
Men	75	12.5
Women	55	10.5

(i) Two men are chosen at random. Find the probability that one of the men has mass more than 90 kg and the other has mass less than 90 kg. [4]

(ii) One man and one woman are chosen at random. Find the probability that the woman's mass is greater than the man's. [4]

The safety limit for a hotel elevator is 530 kg.

(iii) Six men are chosen at random. Find the probability that their total mass is greater than 530 kg. [4]

(iv) Six male hotel guests enter the elevator, at a time when a large number of sumo wrestlers are staying at the hotel. Give two reasons why the probability that their total mass exceeds 530 kg may be different from the value calculated in part **(iii)**. [2]