

MINISTRY OF EDUCATION, SINGAPORE  
in collaboration with  
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE  
General Certificate of Education Advanced Level  
Higher 1

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## MATHEMATICS

8863/01

Paper 1

October/November 2008

3 hours

Additional Materials: Answer Paper  
Graph paper  
List of Formulae (MF15)

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### READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

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This document consists of 5 printed pages and 3 blank pages.



Singapore Examinations and Assessment Board



UNIVERSITY of CAMBRIDGE  
International Examinations

## Section A: Pure Mathematics [40 marks]

- 1 Sketch the graph of  $y = \sin x$  for  $0 \leq x \leq 4\pi$ . [1]

It is given that  $\alpha$  is an acute angle, and  $\sin \alpha = c$ . State, in terms of  $c$ , the value of

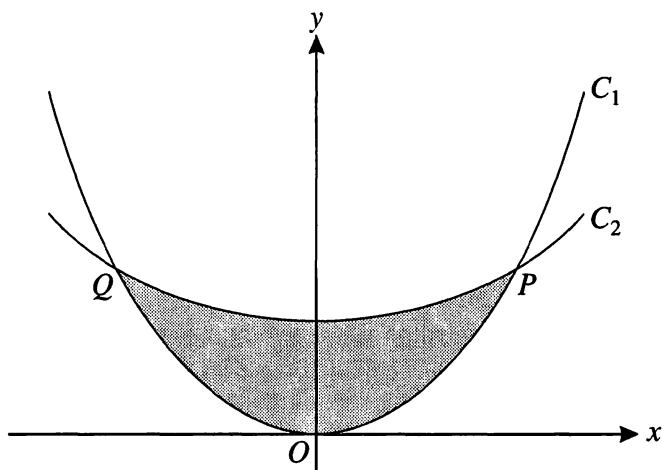
(i)  $\sin(2\pi + \alpha)$ , [1]

(ii)  $\sin(3\pi + \alpha)$ . [1]

State, in terms of  $\alpha$  and  $\pi$ , one value of  $x$  between  $\pi$  and  $2\pi$  for which  $\sin x = -c$ . [1]

- 2 The sum of two numbers  $x$  and  $y$  is 20 and the sum of their squares is 300. Given that  $x > y$ , find the exact values of  $x$  and  $y$ . [5]

3



The diagram shows the graphs of

$$C_1 : y = 2x^2 \quad \text{and} \quad C_2 : y = x^2 + k^2,$$

where  $k$  is a positive constant. The graphs intersect at  $P$  and  $Q$ , as shown. Show that the  $x$ -coordinates of  $P$  and  $Q$  are  $k$  and  $-k$  respectively. [1]

Find the exact value of the area of the shaded region between  $C_1$  and  $C_2$ . [5]

4 The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f : x \mapsto x^2 - 1,$$

$$g : x \mapsto |x|.$$

(i) Sketch, on separate diagrams, the graphs of

(a)  $y = f(x)$ ,

(b)  $y = gf(x)$ ,

(c)  $y = fg(x)$ .

Label each diagram clearly, and mark any intersections of each graph with the axes. [3]

(ii) State the greatest possible value of  $a$  such that the function  $f$ , with domain restricted to  $x \leq a$ , has an inverse,  $f^{-1}$ . [1]

With this value of  $a$  state the domain of  $f^{-1}$ , and find an expression for  $f^{-1}(x)$ . [3]

5 A spot of light on a computer screen moves in a horizontal line across the screen. At time  $t$  seconds, its distance,  $x$  mm, from the left-hand edge of the screen is given, for  $t \geq 0$ , by

$$x = t^3 - 12t^2 + kt,$$

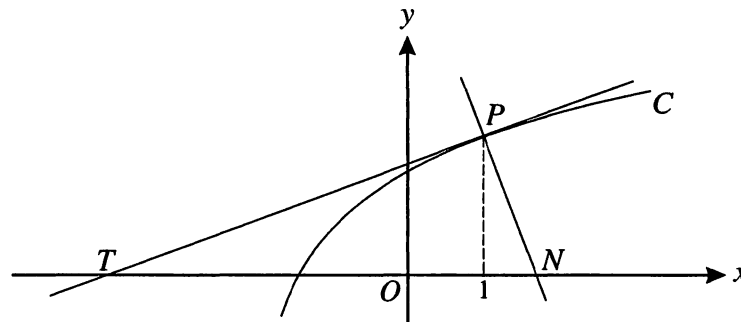
where  $k$  is a positive constant. Find the set of values of  $k$  for which  $x$  is an increasing function of  $t$ . [5]

It is now given that  $k = 36$ .

(i) Sketch the graph of  $x$  against  $t$ . [1]

(ii) The screen has width 375 mm. Find the time in seconds at which the spot reaches the right-hand edge of the screen, giving your answer correct to 1 decimal place. [2]

6



The diagram shows the curve  $C$  with equation

$$y = \ln(2x + 4).$$

The point  $P$  on  $C$  has coordinates  $(1, \ln 6)$ . The tangent to  $C$  at  $P$  meets the  $x$ -axis at  $T$ . Show that the exact  $x$ -coordinate of  $T$  is  $1 - 3 \ln 6$ . [4]

The normal to  $C$  at  $P$  meets the  $x$ -axis at  $N$ . Find the exact  $x$ -coordinate of  $N$ . [2]

Find the exact area of triangle  $PTN$ . [4]

**Section B: Statistics [60 marks]**

- 7 An examination is marked out of 100. It is taken by a large number of candidates. The mean mark, for all candidates, is 72.1, and the standard deviation is 15.2. Give a reason why a normal distribution, with this mean and standard deviation, would not give a good approximation to the distribution of marks. [1]

A random sample of 50 of the candidates is taken. Calculate the probability that the mean mark of this sample lies between 70.0 and 75.0. [3]

- 8 A baker makes loaves of bread. 60% of the loaves that he makes are 'crusty'. A customer buys six randomly chosen loaves. Find the probability that exactly three of them are crusty. [2]

A market trader buys 40 randomly chosen loaves. Use a suitable approximation to find the probability that at least 20 of them are crusty. [4]

The mass of a loaf has a normal distribution with mean 1.24 kg and standard deviation  $\sigma$  kg. The probability that a randomly chosen loaf has mass less than 1 kg is 0.04. Find the value of  $\sigma$ . [3]

- 9 Two children, Tan and Mui, are each to be given a pen from a box containing 3 red pens and 5 blue pens. One pen is chosen at random and given to Tan. A green pen is then put in the box. A second pen is chosen at random from the box and given to Mui.

(i) Draw a tree diagram to represent the possible outcomes. [2]

(ii) Write down the conditional probability that Mui's pen is blue, given that Tan's pen is red. [1]

(iii) Find the probability that Mui's pen is red. [2]

(iv) Find the conditional probability that Tan's pen is red, given that Mui's pen is blue. [5]

- 10 A consumer association is testing the lifetime of a particular type of battery that is claimed to have a lifetime of 150 hours. A random sample of 70 batteries of this type is tested and the lifetime,  $x$  hours, of each battery is measured. The results are summarised by

$$\Sigma x = 10\,317, \quad \Sigma x^2 = 1\,540\,231.$$

The population mean lifetime is denoted by  $\mu$  hours. The null hypothesis  $\mu = 150$  is to be tested against the alternative hypothesis  $\mu < 150$ . Find the  $p$ -value of the test and state the meaning of this  $p$ -value in the context of the question. [5]

A second random sample of 50 batteries of this type is tested and the lifetime,  $y$  hours, of each battery is measured, with results summarised by

$$\Sigma y = 7331, \quad \Sigma y^2 = 1\,100\,565.$$

Combining the two samples into a single sample, carry out a test, at the 10% significance level, of the same null and alternative hypotheses. [6]

- 11 An engineering company makes cranes. The numbers,  $x$ , sold in each three-month period for two years, together with the profits,  $y$  thousand dollars, on the sale of these cranes are given in the following table.

$x$	15	17	13	21	16	22	14	18
$y$	290	350	270	430	340	410	300	360

- (i) Give a sketch of the scatter diagram for the data as shown on your calculator. [2]
- (ii) Find  $\bar{x}$  and  $\bar{y}$ , and mark the point  $(\bar{x}, \bar{y})$  on your scatter diagram. [2]
- (iii) Calculate the equation of the regression line of  $y$  on  $x$ , and draw this line on your scatter diagram. [2]
- (iv) Calculate the product moment correlation coefficient, and comment on its value in relation to your scatter diagram. [2]
- (v) For the next three-month period, the sales target is 20 cranes. Estimate the corresponding profit. [2]
- (vi) The company's sales director uses the regression line in part (iii) to predict the profit if 40 cranes were to be sold in a three-month period. Comment on the validity of this prediction. [2]
- 12 A supermarket obtains a large supply of apples of a single variety. The mass of an apple has a normal distribution with mean 0.234 kg and standard deviation 0.025 kg. Some of the apples are packed, at random, into 'small' bags, each containing 5 apples, and others are packed, at random, into 'large' bags, each containing 10 apples. Find the probability that a randomly chosen small bag has a mass exceeding 1.2 kg. [4]
- Find the probability that the total mass of two randomly chosen small bags is within  $\pm 0.2$  kg of the mass of a randomly chosen large bag. [4]
- Lee buys two small bags at \$1.50 per kg, and Foo buys one large bag at \$1.20 per kg. Find the probability that Lee pays at least \$0.50 more than Foo. [6]

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