MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 1

MATHEMATICS

8863/01

Paper 1

October/November 2009

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Singapore Examinations and Assessment Board

UNIVERSITY of CAMBRIDGE International Examinations

Section A: Pure Mathematics [40 marks]

1 Without using a calculator, solve the simultaneous equations

$$x + 2y = 3,$$
$$x^2 + xy = 2.$$ [4]

2 **(i)** Sketch the graphs of $y = \sqrt{x}$ and $y = \frac{1}{2}x$ on a single diagram and write down the coordinates of the points where $y = \sqrt{x}$ and $y = \frac{1}{2}x$ intersect. [2]

(ii) Find $\int \sqrt{x}\,\mathrm{d}x$ and $\int \frac{1}{2}x\,\mathrm{d}x$. [2]

(iii) Without using a calculator, find the area of the region between the two graphs. [2]

3 The functions f and g are defined by

$$\mathrm{f} : x \mapsto \mathrm{e}^x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto x + 2, \quad x \in \mathbb{R},\ x > -2.$$

The function h is defined by $\mathrm{h} = \mathrm{f}^{-1}\mathrm{g}$.

(i) Find an expression for $\mathrm{h}(x)$. [2]

(ii) Sketch the graph of $y = \mathrm{h}(x)$, stating the equations of any asymptotes and the exact coordinates of any points where the graph crosses the axes. [3]

(iii) Solve the equation $\mathrm{h}(x) = \mathrm{g}(-x)$, giving the root correct to 3 decimal places. [2]

4 **(i)** Sketch the curve $y = x - \frac{1}{x}$, stating clearly the coordinates of all points of intersection with the axes. [1]

(ii) Find the gradient of the normal at the point P on the curve where $x = 2$. [2]

(iii) Find the equation of the normal at P in the form $ax + by + c = 0$, where a and b are integers. [3]

(iv) The normal at P meets the y-axis at N and the tangent at P meets the y-axis at T. Find the area of triangle PTN. [5]

5 A curve has equation $y = 2x^3 - 5x^2 - 4x + 3$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Hence find the exact coordinates of the stationary points on the curve. [4]

(ii) Sketch the curve, stating clearly the coordinates of all points of intersection with the axes. [3]

(iii) Solve the inequality $2x^3 - 5x^2 - 4x + 3 > 0$. Hence find the exact solutions of the inequality $2\mathrm{e}^{3x} - 5\mathrm{e}^{2x} - 4\mathrm{e}^x + 3 > 0$. [5]

Section B: Statistics [60 marks]

6 Three researchers, A, B and C, share an office. When the office phone rings, the probabilities of the call being for each of them are as follows.

$$A : 0.2 \qquad B : 0.3 \qquad C : 0.5$$

The probabilities of each researcher being in the office when the phone rings are as follows.

$$A : 0.7 \qquad B : 0.6 \qquad C : 0.8$$

All the probabilities are independent. Find the probability that, when the phone rings,

(i) the call is for A and A is in the office, [1]

(ii) the researcher being called is in the office, [2]

(iii) the call is for C, given that the researcher being called is not in the office. [2]

7 A and B are two events such that $\mathrm{P}(A) = \frac{1}{3}$, $\mathrm{P}(B) = \frac{2}{5}$ and $\mathrm{P}(A \cup B) = \frac{17}{30}$.

(i) Find $\mathrm{P}(A \cap B)$. [1]

(ii) Show that A and B are not independent. [1]

(iii) Using a Venn diagram, or otherwise, find $\mathrm{P}(A' \cup B)$. [3]

8 Components in machines used in a factory wear out and need to be replaced. The lifetime of a component has a normal distribution with mean 120 days and standard deviation 18 days.

(i) Find the probability that the lifetime of a component is more than 144 days. [2]

(ii) Two components are chosen at random. Find the probability that one has a lifetime of more than 144 days and one has a lifetime of less than 144 days. [2]

A company develops a new design for the component. The standard deviation of the lifetimes remains 18 days, but the company claims that the mean lifetime is longer than for the old components. From a random sample of 50 components of the new design, the sample mean is 124 days. Test at the 5% level of significance whether there is sufficient evidence to support the company's claim. [4]

[Turn over

9 A liquid nutrient is added to the soil around the fruit trees in an orchard, with the aim of increasing the total weight of fruit produced by the trees. For each of 8 trees, the volume of liquid nutrient, $x\,\text{cm}^3$, and the corresponding weight, $y\,\text{kg}$, of fruit per tree is given in the table below.

x	0	20	40	60	90	120	160	200
y	15.1	15.7	16.2	16.8	16.7	16.5	17.3	18.1

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Calculate the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Calculate the equation of the regression line of y on x. Sketch this line on your scatter diagram. [2]

(iv) Estimate the weight of fruit on a tree when $135\,\text{cm}^3$ of liquid nutrient is added to its soil. [1]

(v) Explain why it might be unsuitable to use the equation in part **(iii)** to estimate how much liquid nutrient would be needed for a tree to yield 20 kg of fruit. [1]

10 Over a long period of time, it is found that 20% of candidates who take a particular piano examination fail the examination.

(i) Find the probability that, in a group of 10 randomly chosen candidates who take the examination, exactly 2 will fail. [2]

(ii) It is given that 15% of the candidates who **pass** the piano examination are awarded a distinction. Find the probability that, in a randomly chosen group of 10 candidates who take the examination, fewer than 2 will be awarded a distinction. [3]

(iii) Use a suitable approximation to estimate the probability that, in a group of 50 randomly chosen candidates who take the examination, at most 12 will fail. You should state the mean and variance of the distribution used in the approximation. [4]

11 **(a)** An insurance company receives a large number of claims for flood damage. On a particular day the company receives 72 such claims. Because of staff shortages, it is only possible to process 8 of these claims.

(i) Describe how you would choose a systematic random sample of size 8 from the received claims. [2]

(ii) Comment on whether this method of sampling gives a better indication of the value of the 72 claims as compared to simply choosing as the sample the first 8 claims received. [1]

(b) From the claims received by the company, over a long period of time, a random sample of 120 is taken. The values of the claims, $\$x$, are summarised by

$$\Sigma(x - 1000) = 5320, \qquad \Sigma(x - 1000)^2 = 8\,282\,000.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) What do you understand by the term 'unbiased estimate'? [1]

(iii) The population mean is denoted by $\$\mu$. Using the sample data, a significance test of the null hypothesis $\mu = 1000$ against the alternative hypothesis $\mu \neq 1000$ is carried out at the $\alpha\%$ level of significance. Find the set of values of α for which the null hypothesis will be rejected. [5]

12 **(a)** The plums sold by a supermarket are graded 'small', 'medium' or 'large'. The masses of the plums have a normal distribution. Plums with a mass less than 22 grams are graded as small, plums with a mass greater than 29 grams are graded as large and the rest are graded as medium. Given that 30% of plums are small and 20% are large, find the mean and standard deviation of the distribution. [4]

(b) The masses, in kilograms, of apples and nectarines sold by the supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Apples	0.15	0.03
Nectarines	0.07	0.02

(i) Two apples and four nectarines are chosen at random. Find the probability that the total mass of the two apples is greater than the total mass of the four nectarines. [4]

(ii) Apples cost \$9 per kilogram and nectarines cost \$12 per kilogram. Find the mean and the variance of the total cost of two apples and four nectarines and hence find the probability that the total cost is between \$5 and \$6. [5]

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