§10a. Vectors 1: Extra Practice Questions

February 25, 2017

1. Given $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$,
   (a) Find the magnitude of $\mathbf{a}$
   (b) Write down a unit vector parallel to $\mathbf{a}$
2. A parallelogram $OACB$ is formed by vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{a}$,
   (a) Write $\overrightarrow{AC}$, $\overrightarrow{CB}$, $\overrightarrow{OC}$ and $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
   (b) Express $\overrightarrow{OC} \cdot \overrightarrow{AB}$ in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$.
   (c) If $OACB$ is a rhombus, what can you say about $|\mathbf{a}|$ and $|\mathbf{b}|$?
   (d) Use (b) and (c) to comment on the relationship of the diagonals of a rhombus.
3. What can you conclude about vectors $\mathbf{a}$ and $\mathbf{b}$ if
   (a) $\mathbf{a} \cdot \mathbf{b} < 0$
   (b) $\mathbf{a} \cdot \mathbf{b} = 0$
   (c) $\mathbf{a} \cdot \mathbf{b} > 0$
4. Find the angle between $\mathbf{a} = -2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$.
5. Find the value(s) of k for which $(k\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ and $(4\mathbf{i} + k\mathbf{j} + 3k\mathbf{k})$ are
   (a) parallel to each other,
   (b) perpendicular to each other.
6. (a) Suppose $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors such that $|\mathbf{a} - \mathbf{b}| = |\mathbf{a} + \mathbf{b}|$, Deduce that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
   (HINT: Square both sides to get $|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a} + \mathbf{b}|^2$. Use the fact that $|\mathbf{q}|^2 = \mathbf{q} \cdot \mathbf{q}$ and use vector expansion.)
   (b) A parallelogram $OACB$ is formed by vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
      i. Express $\overrightarrow{OC}$ and $\overrightarrow{BA}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
      ii. What does $|\mathbf{a} - \mathbf{b}| = |\mathbf{a} + \mathbf{b}|$ mean in context of the parallelogram $OACB$?
      iii. What does part (a) tell us about the parallelogram $OACB$?

7. $\mathbf{a} = \mathbf{i} + 3\mathbf{j} + k\mathbf{k}$
   $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$
   $\mathbf{c} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

   If $\mathbf{b}$ is perpendicular to $\mathbf{c} - 2\mathbf{a}$, find the value of $\mathbf{k}$.

8. (a) Given $A(3, -1, 5)$, $B(2, 0, -3)$ and $C(1, 3, -3)$, find $\angle BAC$.

   (b) Find the area of $\triangle ABC$.

9. $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$
   $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
   $\mathbf{c} = -4\mathbf{j} - \mathbf{k}$

   Find $(\mathbf{b} \times \mathbf{c}) \cdot 2\mathbf{a}$

10. The position vectors of A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. If P lies on line AB such that $\overrightarrow{AP} = t\overrightarrow{AB}$, find the position vector of P in terms of A and B.

11. $A(2, -1, 4)$, $B(-3, 1, -1)$

    Find P on line AB such that $AP : PB = 2 : 5$

12. $ABCD$ is a parallelogram with $A(-1, 2, 3)$, $B(0, 2, 4)$ and $C(1, 5, -5)$.

    (a) Find the coordinates of D.

    (b) Find the area of $ABCD$.

13. A cuboid is formed starting at the origin using the vectors $2\mathbf{i}$, $3\mathbf{j}$, $5\mathbf{k}$. One diagonal is formed by joining the origin (bottom left corner) to the upper right corner. Another diagonal is formed by joining the bottom right corner to the upper left corner.

    Find the acute angle between its two diagonals.

14. Given vectors $\mathbf{a}$ and $\mathbf{b}$, with $|\mathbf{a}| = 2$, find the value of $|\mathbf{a} + 2\mathbf{b}|$ if

    (a) $\mathbf{b} = -2\mathbf{a}$

    (b) $\mathbf{a}$ and $\mathbf{b}$ are perpendicular and $|\mathbf{b}| = 6$.

    (HINT: Use the fact that $|\mathbf{q}|^2 = \mathbf{q} \cdot \mathbf{q}$.)

15. Points A, B, C, and D have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively. M is the midpoint of line segment AB while N is the midpoint of line segment BC.

    (a) Express $\overrightarrow{MN}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$

    P and Q are the midpoints of line segments CD and DA respectively.

    (b) By finding an expression for $\overrightarrow{QP}$ in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$, show that $MNPQ$ is a parallelogram.

16. Three points have coordinates $A(4, 1, 2)$, $B(1, 5, 1)$ and $C(\lambda, \lambda, 3)$.

    (a) Find the value of λ for which the triangle ABC has a right angle at B.

    (b) For this value of λ, find the coordinates of point D on side AC such that $AD = 2DC$.

Answers

1. 3.
   $\frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$.

2. $\mathbf{b}, -\mathbf{a}, \mathbf{a} + \mathbf{b}, \mathbf{b} - \mathbf{a}$.
   $|\mathbf{b}|^2 - |\mathbf{a}|^2$.
   $|\mathbf{a}| = |\mathbf{b}|$.
   The diagonals are perpendicular.

3. The vectors make an obtuse angle with each other.
   The vectors are perpendicular or $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$.
   The vectors make an actue angle with each other.

4. 108.8°.

5. $k = \pm 2$.
   $k = 0$.

6. $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$.
   The diagonals have the same length.
   The parallelogram is a rectangle.

7. $-\frac{1}{5}$.

8. 19.9°.
   12.7 units^2.

9. 16.

10. $(1 - t)\mathbf{a} + t\mathbf{b}$.

11. $P(\frac{4}{7}, \frac{-3}{7}, \frac{18}{7})$.

12. $D(0, 5, -6)$.
    $2\sqrt{6}$.

13. 37.9°.

14. 36. 28.

15. $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$.
    $\overrightarrow{QP} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c} = \overrightarrow{MN}$.

16. $\lambda = 19$.
    $D(14, 13, \frac{8}{3})$.