Module 6C: Statistics (Normal Distribution and Sampling Distribution)

1. [2015/NYJC/II/6]

Chickens sold by a supermarket are graded 'small', 'medium' or 'large'. The weights of the chicken have a normal distribution. Chickens with weight less than 1 kg are graded as 'small', chickens with weight greater than 1.8 kg are graded as 'large' and the rest are graded as 'medium'. Given that 20% of chicken are small and 15% are large, find the mean and standard deviation of the distribution. [4]

2. [2015/TPJC/II/7]

A hair salon offers two hair care treatment packages, namely the Herbal Treatment and the Wonder Treatment. The length of time taken for one Herbal Treatment session has a normal distribution with mean μ minutes and standard deviation 5 minutes. The length of time taken for one Wonder Treatment session has an independent normal distribution with mean 50 minutes and standard deviation 7 minutes.

(i) Alice went for a session of Herbal Treatment while Betty went for a session of Wonder Treatment. The probability that Alice has to wait for Betty for at least five minutes after her own session is 0.05. Assume that they started their sessions at the same time, show that the value of μ is 59.1, correct to 3 significant figures. [3]

(ii) Using the value of μ shown in part **(i)**, find the probability that the total time taken for 3 sessions of the Herbal Treatment exceeds four times the time taken for 1 session of Wonder Treatment. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

3. [2015/SRJC/II/11]

(a) A farm in the west of Singapore grows turnips for sale to the local market.

(i) Five turnips are randomly chosen. Find the probability that exactly one turnip weighs less than the lower quartile weight and exactly two turnips weigh more than the median weight. [2]

(ii) The mass of a randomly chosen turnip has mean 40 g and standard deviation of 3 g. If the probability that the mean mass of a large sample of n turnips is greater than 39.6 g exceeds 0.95, find the least value of n. [3]

(b) A random variable X has the distribution $X \sim \mathrm{N}(40, 3^2)$. The random variable Y is related to X by the formula $Y = aX - \frac{1}{b}$, where a and b are constants and $a > 0$. Given that $\mathrm{P}(Y < 85) = \mathrm{P}(Y > 155) = 0.075$, find the values of $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$, and hence find the values of a and b. [5]

4. [2016/IJC/II/6]

Historical data shows that the number of goals scored per match at European Football Championships has a mean of 1.93 and a variance of 1.4. A large random sample of n matches is taken. Find the least value of n such that the probability that the average number of goals scored per match exceeds 2 goals is less than 0.24. [5]

5. **[2016/NYJC/II/6]**

The mass, in grams, of an ice-cube has the distribution $N(\mu, \sigma^2)$. The mean mass of a random sample of *n* ice-cubes is denoted by $\overline{X}$. It is given that $P(\overline{X} < 35.0) = 0.97725$ and $P(\overline{X} \geq 20.0) = 0.84134$.

(i) Obtain an expression for σ in terms of *n*. [3]

(ii) Find $P(\overline{X} > 32)$. [2]

Assume now that the mass of an ice-cube has the distribution $N(25, 50)$.

An ice dispenser discharges 15 ice cubes each time into a cup. State the distribution of the mass of a discharge of 15 ice cubes. [1]

(iii) Find the mass exceeded by 10% of these discharges, correct to 1 decimal place. [2]

(iv) Find the probability that the mass of the first discharge of ice-cubes is more than the second discharge. [2]

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Answers:

1. $\mu = 1.36$ and $\sigma = 0.426$

2. **(ii)** 0.219

3. **(a)(i)** 0.117 **(ii)** 153

(b) $a = 8.10$; $b = 0.00490$

4. 143

5. **(i)** $\sigma = 5\sqrt{n}$ **(ii)** 0.0808

(iii) $a = 410.1$ **(iv)** 0.5

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