CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Mid-Year Examination

MATHEMATICS **9758/01**

Paper 1 **10 May 2018**

3 hours

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER.
Place the entire question paper in front and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Name: ____________________________ **Class:** ________________

Question	1	2	3	4	5	6	7	8	9	10	Total
Marks											
Total	6	7	10	8	12	10	13	10	13	11	100

This document consists of **6** printed pages, including the cover page.

Section A: Pure Mathematics [66 marks]

1 By successively differentiating $e^x \tan x$, find the Maclaurin series for $e^x \tan x$, up to and including the term in x^2.

Hence find the Maclaurin's series for $\dfrac{e^x \tan x}{(1+x)^2}$ up to and including the term in x^2. [6]

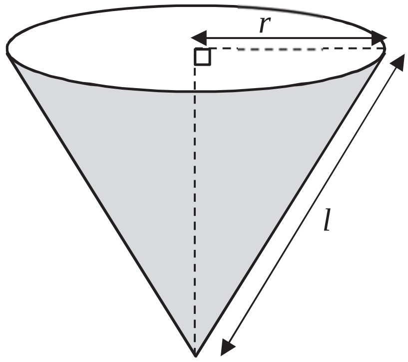
2 **(i)** Find $\int x \sin kx \, dx$, where k is a positive constant. [2]

(ii) Find the constants A, B and C such that $\cos 5x \sin 3x = A(\sin Bx + \sin Cx)$. [2]

(iii) Hence find $\int 3x \cos 5x \sin 3x \, dx$. [3]

3 [It is given that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$ and the curved surface area is $\pi r l$ where l is the slant height of the cone.]

An open inverted conical trough is made of a metal sheet of fixed area of 9 m^2 with negligible thickness. It is given that the conical trough has radius r m and a slant height of l m as shown in the diagram.



(i) Show that the volume V of the conical trough is given by $V = \frac{1}{3} r \sqrt{81 - \pi^2 r^4}$. [2]

(ii) Determine the exact radius r which gives the maximum value of V, proving that it is a maximum. [4]

It is given that $r = 2$ m and $l = 2\sqrt{5}$ m. Water is pumped into the empty trough at a constant rate of $0.02 \text{ m}^3/\text{min}$.

(iii) Find the rate at which the height of the water level is increasing at the instant when the height of the water level is 1 m. [4]

4 A curve C has parametric equations

$$x = t + \sin t, \qquad y = 1 - \cos t,$$

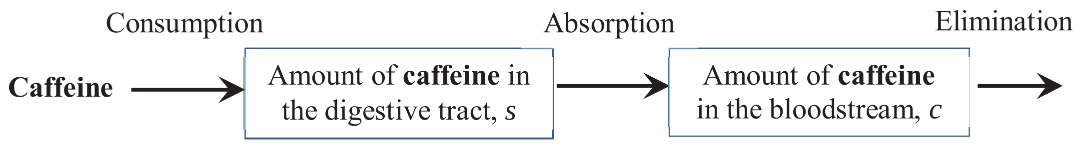
where $-\dfrac{\pi}{4} \le t \le \pi$.

(i) Find the exact equation of the normal to the curve C at $t = \dfrac{\pi}{3}$. [4]

(ii) Find the area of the region bounded by C, the x-axis and the normal to C at $t = \dfrac{\pi}{3}$. [4]

5 When a caffeinated beverage is consumed, caffeine is absorbed from the digestive tract into the bloodstream, and subsequently eliminated from the bloodstream through first order kinetics.

<u>Model used</u>



The amount of caffeine in the digestive tract, s, is absorbed into the bloodstream at a rate proportional to the amount of caffeine remaining in the digestive tract, with a constant of proportionality 2, with respect to time, t hours.

(i) Write down a differential equation relating s and t. [1]

While the amount of caffeine c in the bloodstream increases as caffeine is absorbed from the digestive tract, it is also eliminated from the bloodstream at a rate proportional to the amount of caffeine c in the bloodstream, producing a net rate of change modelled by

$$\frac{dc}{dt} = 2s - \frac{c}{8}$$

(ii) Hence, find in terms of time t, an expression for the amount of caffeine s in the digestive tract, and show that $\dfrac{dc}{dt} = 2s_0\,e^{-2t} - \dfrac{c}{8}$, where s_0 is the initial amount of caffeine in the digestive tract. [3]

(iii) By using the substitution $c = xe^{-\frac{1}{8}t}$, reduce the differential equation derived in part **(ii)** to a differential equation of the form $\dfrac{dx}{dt} = f(t)$.

Given that the amount of caffeine in the bloodstream is initially zero, show that the amount of caffeine in the bloodstream is given by $c = ks_0\,(e^{-2t} - e^{-\frac{1}{8}t})$, where k is a rational number to be determined. [7]

(iv) State one assumption in the model used. [1]

6 It is given that

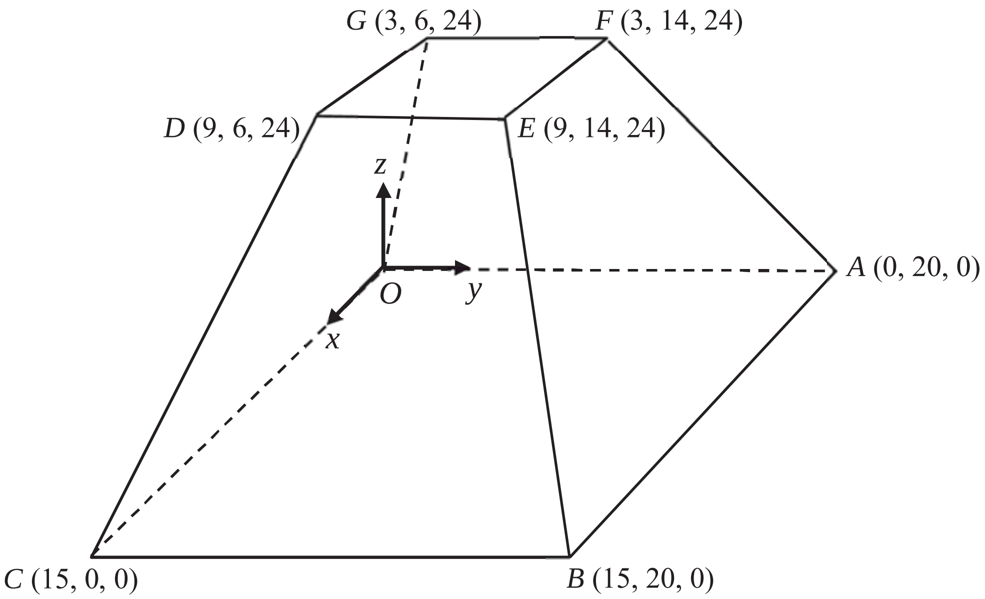
$$z^3 + 2z^2 + \left(k - 8\sqrt{2}\mathrm{i}\right)z + 8 - 4\sqrt{2}k\mathrm{i} = 0,$$

where k is a real constant, has a real root.

(i) Show that -2 is the real root and find the value of k. [4]

(ii) Hence find the other roots, giving your answers in exact cartesian form $a + \mathrm{i}b$. [6]

7 A glass ornament *OABCDEFG* is a truncated pyramid on a rectangular base (see Figure below). All dimensions are in centimetres. The point O is the origin with unit vectors **i** along *OC*, **j** along *OA* and **k** vertically upwards.



(i) Find the cartesian equation of the surface *BCDE*. [4]

Let M be the mid-point of side *FA*.

(ii) Find the shortest distance from B to the line *OM*, giving your answer in exact form. [3]

(iii) Show that the lines *OG* and *AF* meet at the point P with coordinates (5, 10, 40). [4]

(iv) Find the angle between the surface *BCDE* and the base *OABC* of the ornament. [2]

Section B: Probability and Statistics [34 marks]

8 Three students from the School of Arts, four students from the School of Business and five students from the School of Science participate in a social networking activity. The activity requires them to sit at a round table .

(i) Find the number of different possible seating arrangements. [1]

Let A be the event that all students from the School of Arts are separated from one another and B be the event that all students from the School of Business are seated together.

(ii) Find $\mathrm{P}(A)$. [2]

(iii) Find $\mathrm{P}(B \mid A)$. [3]

(iv) Determine if A and B are independent. [2]

The activity organiser decides to give out prizes to n students. Assuming that each student has an equal chance of getting a prize,

(v) find the maximum value of n such that the probability that all prizes are given to students from the School of Science exceeds 0.01. [2]

9 The air pressure, in pounds per square inch (psi), of a basketball is a random variable X with distribution $\mathrm{N}\left(\mu, \sigma^2\right)$.

(i) It is known that $\mathrm{P}(X < 7) = 0.02275$ and $\mathrm{P}(X < 9) = 0.97725$. Write down the value of μ and show that $\sigma = 0.5$. [3]

The air pressure, in psi, of a volleyball is a random variable Y with distribution $\mathrm{N}\left(4.26, 0.8^2\right)$.

(ii) Calculate the probability that a randomly chosen basketball and a randomly chosen volleyball each has air pressure exceeding 6 psi. [3]

(iii) Calculate the probability that the sum of air pressures in a randomly chosen basketball and volleyball exceeds 12 psi. [2]

(iv) Explain why the probability in part **(ii)** is smaller than that in part **(iii).** [1]

(v) Calculate the probability that the sum of air pressures in 6 volleyballs is less than thrice the air pressure in a basketball. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

(vi) State an assumption for your calculations in parts **(ii), (iii)** and **(v)**. [1]

10 A retailer sells 5 grades of washing detergent, Grades *A*, *B*, *C*, *D* and *E*, with Grade *A* being the premium grade. The price of a bottle of detergent sold is denoted by X. The price and the probability at which each grade of detergent is sold are as follows:

Grade	A	B	C	D	E
Price per bottle, \$$x$	22	19	16	13	10
$\mathrm{P}(X=x)$	$\frac{1}{12}$	$\frac{1}{4}$	a	$\frac{1}{3}$	$\frac{1}{6}$

Assuming that the bottles of detergent are sold independently, find

(i) a. [1]

(ii) the expectation and variance of the price of one bottle of detergent. [2]

In order to improve sales, the retailer decides to sell every bottle of detergent with a free towel which comes in different colours. The probability that a bottle of detergent is sold with a pink towel is p.

(iii) It is given that $p = 0.2$.

(a) If a housewife buys 19 bottles of detergent, find the most likely number of bottles with a pink towel. [2]

(b) If N bottles of detergent are packed in a box, find the least value of N such that the probability of getting at least two bottles with a pink towel in a box is more than 0.5. [3]

(iv) For an unknown value of p, it is given that if a housewife buys 20 bottles of detergent, the probability that she gets exactly 10 pink towels is 0.003237, correct to 6 decimal places. By forming an equation in terms of p, find the possible values of p. [3]

— END OF PAPER —

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS
JC2 MID-YEAR EXAMINATION SOLUTIONS 2018

Section A: Pure Mathematics [66 marks]

1	Maclaurin's Series		
	Assessment Objectives	**Solution**	**Feedback**
	Able to differentiate successively and find Maclaurin's Series	$y = e^x \tan x$ Differentiate w.r.t. x: $\frac{dy}{dx} = e^x \sec^2 x + e^x \tan x$ $\frac{dy}{dx} = e^x \sec^2 x + y$ Differentiate w.r.t. x: $\frac{d^2y}{dx^2} = e^x\left(2\sec x \sec x \tan x\right) + e^x \sec^2 x + \frac{dy}{dx}$ $= 2\left(\sec^2 x \tan x\right)e^x + e^x \sec^2 x + \frac{dy}{dx}$ When $x = 0$, $y = 0$ $\frac{dy}{dx} = 1$ $\frac{d^2y}{dx^2} = 2$ $y = 0 + x + \frac{2}{2!}x^2 + \ldots$ $y = x + x^2 + \ldots$	Most of the students did this part easily, though there were many variations in their presentations to find this series. A few students were penalized because they used the results found in MF26, they did not read the question with care (i.e. by successively differentiating …). Surprisingly, quite a significant number of students did not present the working properly or use the notation accurately, for instance, commonly seen presentation $\frac{d}{dx} = e^x \sec^2 x + \tan x, \frac{d^2}{dx^2} = e^x \sec^2 x + \tan x.$ In addition, a surprising number made it quite unbelievably difficult for themselves by writing, for instance, $\frac{dy}{dx} = e^x \sec^2 x + e^x \tan x = \frac{2e^x}{\cos 2x + 1} + \frac{e^x \sin x}{\cos x}$ and differentiating it one more time using quotient rule. These students tend to spend some time to simplify and made unnecessary algebraic slips. Many students did not know how to differentiate $\sec^2 x$ with respect to x and many gave answer as $2\sec x \tan x$.

	Able to apply binomial series expansion and deduce a given Maclaurin's series expression	$\frac{e^x \tan x}{(1+x)^2} = \left(e^x \tan x\right)(1+x)^{-2}$ $= \left(x+x^2\right)\left(1+(-2)x+...\right)$ $= \left(x+x^2\right)\left(1-2x+...\right)$ $= x+x^2-2x^2+...$ $= x-x^2+...$	Again, many students did not read the question carefully, they differentiate the terms and apply the Maclaurin series expansion to get the desired series, instead of using the previous series in the working; they gained no marks.

2	Integration Techniques		
	Assessment Objectives	**Solution**	**Feedback**
	Able to solve Integration by parts.	**(i)** $\int x\sin kx \, dx = -\frac{x\cos kx}{k} + \int \frac{\cos kx}{k} dx$ $= -\frac{x\cos kx}{k} + \frac{1}{k}\left[\frac{\sin kx}{k}\right] + C$ $= \frac{\sin kx}{k^2} - \frac{x\cos kx}{k} + C$	Majority of students knew that this part was to be solved by integration by parts. Commonly seen mistakes were wrong formula used, chose the wrong u and omitted constant k after integration. Many students did not introduce the constant of integration in the last step.
	Able to apply the factor formulaue and apply the results obtained to solve the integral.	**(ii)** $\cos 5x \sin 3x = \frac{1}{2}\left[\sin(5x+3x) - \sin(5x-3x)\right]$ $= \frac{1}{2}(\sin 8x - \sin 2x)$ **(iii)** $\int 3x\cos 5x \sin 3x \, dx = \int 3x\left(\frac{1}{2}\sin 8x - \frac{1}{2}\sin 2x\right) dx$ $= \int \frac{3}{2}x\sin 8x - \frac{3}{2}x\sin 2x \, dx$ $= \frac{3}{2}\int x\sin 8x \, dx - \frac{3}{2}\int x\sin 2x \, dx$ $= \frac{3}{2}\left(\frac{\sin 8x}{8^2} - \frac{x\cos 8x}{8}\right) - \frac{3}{2}\left(\frac{\sin 2x}{2^2} - \frac{x\cos 2x}{2}\right) + C$ $= \frac{3}{16}\left(\frac{\sin 8x}{8} - x\cos 8x\right) - \frac{3}{4}\left(\frac{\sin 2x}{2} - x\cos 2x\right) + C$	Most students gained mark for this part though some algebraic slips in solving the unknown constants. Most of the students knew that they were to use previous part(s) to do. However, after substituting the cos5xsin3x , quite a number of students used integration by parts again without realising that the form obtained was similar to part **(i)**. Also, some tend to omit the x terms in the working. Many students did not have the good habit to simplify answers and introduce the constant of integration.

3	Applications of Differentiation		
	Assessment Objectives	**Solution**	**Feedback**
	Able to formulate the expression of curved surface area of cone and slant height (Pythagoras Thereom) and make h the subject. Express volume of cone V in terms of r only.	**(i)** $\pi rl = 9$ $\pi r\sqrt{r^2+h^2} = 9$ $\sqrt{r^2+h^2} = \frac{9}{\pi r}$ $r^2+h^2 = \left(\frac{9}{\pi r}\right)^2$ $h^2 = \frac{81}{\pi^2 r^2} - r^2$ $= \frac{81-\pi^2 r^4}{\pi^2 r^2}$ $h = \sqrt{\frac{81-\pi^2 r^4}{\pi^2 r^2}}$ $= \frac{\sqrt{81-\pi^2 r^4}}{\pi r}$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi r^2\left(\frac{\sqrt{81-\pi^2 r^4}}{\pi r}\right)$ $= \frac{1}{3} r\sqrt{81-\pi^2 r^4}$	Most students were able to identify the curved surface area πrl and equate to 9, making l the subject. Generally, students can prove the given volume equation with the exception of a significant majority who made the following mistakes: 1. Could not handle algebraic manipulation: × $h = \sqrt{\frac{81-\pi^2 r^4}{\pi^2 r^2}} = \sqrt{81-\pi^2 r^4}$ (unacceptable to ignore denominator) × Skip steps to show $V = \frac{1}{3} r\sqrt{81-\pi^2 r^4}$ correctly and attempt to "smoke" the examiner. 2. Ridiculously show the wrong formula when $V = \frac{1}{3} r\sqrt{81-\pi^2 r^4}$ is given in the question paper. 3. Fatal algebraic error: $\sqrt{81-\pi^2 r^4} = 9 - \pi r$??? (We cannot square root term by term this way!) 4. Some students do not even bother to show working until they reach the show result.
	Able to differentiate given expression and find stationary value. Able to prove that V is a maximum by first/ second derivative test	**(ii)** $V = \frac{1}{3} r\sqrt{81-\pi^2 r^4}$	Again, since formula is given in part (i), copy the correct formula and start to differentiate – this is evident throughout the cohort. Avoid careless mistakes please! Students generally know the standard procedure to solve: 1. Find $\frac{dV}{dr}$ in terms of r

$$\frac{dV}{dr}=\frac{1}{3}\sqrt{81-\pi^2r^4}+\frac{1}{3}r\cdot\frac{1}{2}\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-4\pi^2r^3\right)$$

$$=\frac{1}{3}\sqrt{81-\pi^2r^4}+\frac{1}{3}\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-2\pi^2r^4\right)$$

$$=\frac{1}{3}\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(81-\pi^2r^4-2\pi^2r^4\right)$$

$$=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(27-\pi^2r^4\right)$$

At stationary point, $\frac{dV}{dr}=0$

$$\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(27-\pi^2r^4\right)=0$$

$$\pi^2r^4=27$$

$$r^4=\frac{27}{\pi^2}$$

$$r=\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}$$

To prove that V is a maximum:
1st derivative test

r	$r=\left(\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}\right)^-$ $=1.28$	$r=\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}$ $=1.286$	$r=\left(\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}\right)^+$ $=1.29$
$\frac{dV}{dr}$	>0	0	<0
Tangent slope	/	—	\

2. Let $\frac{dV}{dr}=0$ and solve for r
3. Using 1st or 2nd derivative test, prove that V is a maximum.

Common algebraic slips and mistakes include:

1. Chain rule is forgotten here!
   ("–" and $4\pi^2$ are commonly neglected)
2. When simplifying, this r is commonly forgotten to be multiplied to get r^4 and hence, it affects the ability to solve for r later on.
3. Difficulty handling $\sqrt{81-\pi^2r^4}$ and $\left(81-\pi^2r^4\right)^{-\frac{1}{2}}$ when simplifying.
   At this step: $\sqrt{81-\pi^2r^4}=2\pi^2r^4\frac{1}{\sqrt{81-\pi^2r^4}}$,
   students tend to cancel the common $\sqrt{81-\pi^2r^4}$ instead of doing cross multiplication.
4. When rearranging terms, forgot to change – to +. If students are not able to arrive at a sensible answer for r say r is -ve, they do not go back to check their errors and left it undone. This is not acceptable!
5. A handful gave r as decimal when question asked for exact.

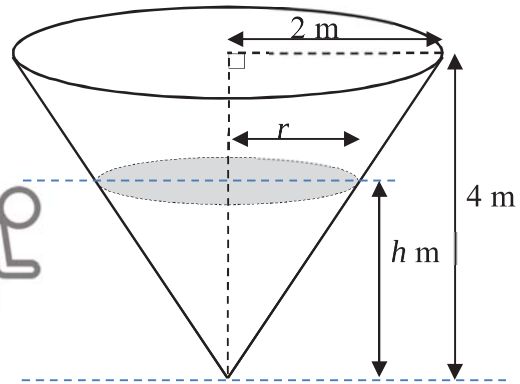
Students ought to prove that V is a maximum but showing clearly

Either 1st derivative test:

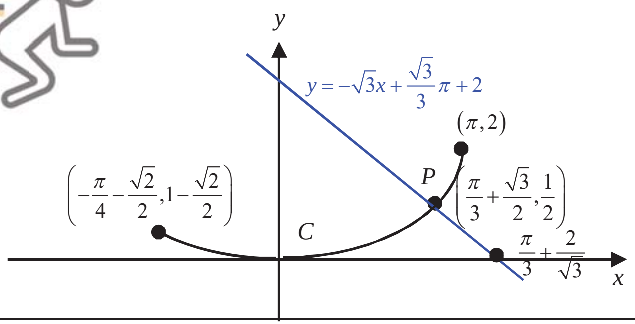
- Headers i.e. values of r must be shown clearly. Use ± 0.01 is safer than values too far apart.

Or 2nd derivative test:

- Not recommended as it involves rather tedious algebraic manipulation.

		V is a maximum when $r=\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}$ 2nd derivative test $\frac{dV}{dr}=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(27-\pi^2r^4\right)$ $\frac{d^2V}{dr^2}=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-4\pi^2r^3\right)+\left(27-\pi^2r^4\right)\left(-\frac{1}{2}\right)\left(81-\pi^2r^4\right)^{-\frac{3}{2}}\left(-4\pi^2r^3\right)$ $=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-4\pi^2r^3\right)+\left(27-\pi^2r^4\right)\left(81-\pi^2r^4\right)^{-\frac{3}{2}}\left(2\pi^2r^3\right)$ When $r=\left(\frac{27}{\pi^2}\right)^{\frac{1}{4}}\Rightarrow r^4=\frac{27}{\pi^2}$ $\frac{d^2V}{dr^2}$ $=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-4\pi^2r^3\right)+\left(27-\pi^2r^4\right)\left(-\frac{1}{2}\right)\left(81-\pi^2r^4\right)^{-\frac{3}{2}}\left(-4\pi^2r^3\right)$ $=\left(81-\pi^2r^4\right)^{-\frac{1}{2}}\left(-4\pi^2r^3\right)<0$ (maximum)	- Value or sign of $\frac{d^2V}{dr^2}$ must be clearly shown. Some students evaluate the maximum *V* but it is not required in the question.
	Able to solve rate of change problems 	**(iii)**  Fig. 2	Majority of the students are not able to answer this question. If you had read the question carefully, the concept here involves identifying *V* as volume of the water at time *t* instead of volume of trough in part (i). Candidates should not use the formula given in part (i) as they are referring to two different quantities. Common procedural mistakes include: 1. Substituting $h=1$ in $V=\frac{1}{3}\pi r^2h$ 2. Writing $\frac{r}{1}=\frac{2}{4}\Rightarrow r=\frac{1}{2}$ straight away. 3. Defining $h=\sqrt{\left(2\sqrt{5}\right)^2-r^2}$ but all *r*, *h* and *l* vary with time.

		$l^2 = r^2 + h^2$ $\left(2\sqrt{5}\right)^2 = 2^2 + h^2$ $h = 4$ By similar triangles, $\frac{r}{h} = \frac{2}{4} \Rightarrow r = \frac{1}{2}h$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $= \frac{\pi h^3}{12}$ $\frac{dV}{dh} = \frac{3\pi h^2}{12}$ $= \frac{\pi h^2}{4}$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $0.02 = \frac{\pi(1)^2}{4}\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{0.02}{\frac{\pi}{4}}$ $= 0.02546479$ ≈ 0.0255 m/min (3 s.f.)	4. Treating r or h as a constant but both r and h are functions in terms of h. Students should draw the diagram and visualize. Do not substitute the values of h straight away as V and r changes with h all the time.

4	Definite Integrals		
	Assessment Objectives	**Solution**	**Feedback**
	Able to perform parametric differentiation. Able to find the equation of normal.	**(i)** $\frac{dy}{dx}=\frac{\sin t}{1+\cos t}$ When $t=\frac{\pi}{3}$, $x=\frac{\pi}{3}+\sin\frac{\pi}{3}=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$, $y=1-\cos\frac{\pi}{3}=\frac{1}{2}$ $\therefore P\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2},\frac{1}{2}\right)$. $-\frac{1}{\frac{dy}{dx}}=-\frac{1+\cos t}{\sin t}$ When $t=\frac{\pi}{3}$, $-\frac{1}{\frac{dy}{dx}}=-\frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\sqrt{3}$ <u>Equation of the normal @ *P*</u> $y-\frac{1}{2}=-\sqrt{3}\left(x-\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ $y=-\sqrt{3}x+\frac{\sqrt{3}}{3}\pi+2$	Some students tried forming Cartesian equation of curve before differentiating (with some even substituting $t=\frac{\pi}{3}$ while doing so!). We need to remind them that it is not always easy/useful to do so. Many students could not recall the exact values of trigonometric ratios of special angles such as $\sin\frac{\pi}{3}$. They were heavily penalized for this in this question. Students are expected and should be reminded to simplify their final answer to its simplest form and not leave the examiner to figure out expressions like "$y=\frac{3}{\sqrt{3}}x+\frac{3\pi}{3\sqrt{3}}+\frac{3\sqrt{3}}{2\sqrt{3}}+\frac{1}{2}$".
	Able to use the aid of a sketch to find the area bounded by the region described.	**(ii)** 	Quite a number of students did not attempt this part. Many students sketched the graph of the curve wrongly as they blatantly disregard the given limits of *t*. As a result, some of them misinterpreted the described area as a wrong one.

$$y=-\sqrt{3}x+\frac{\sqrt{3}}{3}\pi+2$$

When $y=0 \Leftrightarrow -\sqrt{3}x+\frac{\sqrt{3}}{3}\pi+2=0$, $x=\frac{\pi}{3}+\frac{2}{\sqrt{3}}$.

Hence,

Area

$$=\int_0^{\frac{\pi}{3}}(1-\cos t)(1+\cos t)\ \mathrm{d}t+\frac{1}{2}\left(\frac{\pi}{3}+\frac{2}{\sqrt{3}}-\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)\frac{1}{2}$$

$$=\int_0^{\frac{\pi}{3}}\sin^2 t\ \mathrm{d}t+\frac{1}{2}\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}\right)\frac{1}{2}$$

$$=\int_0^{\frac{\pi}{3}}\frac{1-\cos 2t}{2}\ \mathrm{d}t+\left(\frac{1}{8\sqrt{3}}\right)$$

$$=\left[\frac{1}{2}t-\frac{\sin 2t}{4}\right]_0^{\frac{\pi}{3}}+\left(\frac{1}{8\sqrt{3}}\right)$$

$$=\frac{1}{2}\left(\frac{\pi}{3}\right)-\frac{\sqrt{3}}{8}+\left(\frac{1}{8\sqrt{3}}\right)$$

$$=\frac{\pi}{6}-\frac{1}{8\sqrt{3}}$$

$$=\frac{\pi}{6}-\frac{1}{4\sqrt{3}}\ \text{units}^2 \quad \text{or } 0.451\ \text{units}^2$$

5	Differential Equations		
	Assessment Objectives	**Solution**	**Feedback**
	Formulate a differential equation (DE) from description(s) in a contextual problem.	(i) $\frac{\mathrm{d}s}{\mathrm{d}t}=-2s$ <u>Explanation</u> : As the amount of caffeine s in the digestive tract decreases solely from absorption into the bloodstream, $\frac{\mathrm{d}s}{\mathrm{d}t}=-\begin{pmatrix}\text{Absorption}\\ \text{rate}\end{pmatrix}$, where $\begin{pmatrix}\text{Absorption}\\ \text{rate}\end{pmatrix}\propto s$, s being the amount of caffeine remaining in the digestive tract. $\Rightarrow \begin{pmatrix}\text{Absorption}\\ \text{rate}\end{pmatrix}=ks$, k being a const. of proportionality Since $k=2$ (given constant of proportionality), $\begin{pmatrix}\text{Absorption}\\ \text{rate}\end{pmatrix}=2s \Rightarrow \frac{\mathrm{d}s}{\mathrm{d}t}=-2s$.	Only a minority of the candidates (20-30%) could accurately formulate a differential equation relating s and t from the context. Many candidates were unable to recognize that the amount of caffeine remaining in the digestive tract was s itself, and had taken unnecessary steps to find erroneous expressions such as $(s-c)$ for the amount of caffeine remaining in the digestive tract, e.g. writing $\frac{\mathrm{d}s}{\mathrm{d}t}\propto(s-c)$ ✗. Some others had mistaken the amount of caffeine c in the bloodstream for the amount of caffeine s left in the digestive tract, e.g. writing $\frac{\mathrm{d}s}{\mathrm{d}t}\propto c$ ✗ or $\frac{\mathrm{d}s}{\mathrm{d}t}=2c$ ✗. Among candidates who could correctly recognize / point out that $\frac{\mathrm{d}s}{\mathrm{d}t}\propto s$ ✓, many erroneously omitted a negative sign (-) in formulating the DE, e.g. writing $\frac{\mathrm{d}s}{\mathrm{d}t}=2s$ ✗, without recognizing that the rate of change in s (amount of caffeine in the digestive tract) is a rate of <u>**decrease / loss**</u> from the absorption into the bloodstream.

	Solve a differential equation using the method of separation of variables, using initial conditions to derive at a particular solution.	**(ii)** Solving this DE : $\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t}=-2$ $\int \frac{1}{s}\ \mathrm{d}s=\int -2\ \mathrm{d}t$ $\ln\lvert s\rvert=-2t+A,$ where A is an arbitrary constant $\lvert s\rvert=\mathrm{e}^{-2t+A}$ $s=\pm\mathrm{e}^{-2t+A}=B\mathrm{e}^{-2t}$, where $B=\pm\mathrm{e}^{A}$ At initial time $t=0$, $s=s_0$ (initial amount of caffeine in the digestive tract) $s_0=B\mathrm{e}^{-2(0)}$, $B=s_0$. $\therefore\ s=s_0\,\mathrm{e}^{-2t}$. Since $\frac{\mathrm{d}c}{\mathrm{d}t}=2s-\frac{c}{8}$ (given), $\frac{\mathrm{d}c}{\mathrm{d}t}=2s_0\,\mathrm{e}^{-2t}-\frac{c}{8}$ (shown).	Among candidates who could correctly formulate the differential equation (DE) for s and t in part (i), most were able to proceed with solving the DE via the method of separation of variables. Even where the erroneous DE $\frac{\mathrm{d}s}{\mathrm{d}t}=2s$, where a negative sign was omitted, was obtained in part (i), up to 2 marks of credit is still given for a correct method of solving DE thru. separation of variables. The main reason why many candidates were unable to secure any credit from answering this part, could be attributed to incorrect DE formulated in part (i), especially those that erroneously included c, e.g. $\frac{\mathrm{d}s}{\mathrm{d}t}=2(s-c)$, $\frac{\mathrm{d}s}{\mathrm{d}t}=2c$, that can no longer be solved by merely attempting to separate variables.

	Use a given substitution relation to convert an existing DE into another of a specfic form (that's more readily solved). Solve a DE using the method of direct integration. Substituting back to solve original DE, using initial conditions to derive a particular solution.	**(iii)** Considering given substitution relation $c = xe^{-\frac{1}{8}t}$ $\frac{dc}{dt} = \frac{dx}{dt}e^{-\frac{1}{8}t} + x\left(-\frac{1}{8}e^{-\frac{1}{8}t}\right)$. Since $\frac{dc}{dt} = 2s_0 e^{-2t} - \frac{c}{8}$, $\frac{dx}{dt}e^{-\frac{1}{8}t} \cancel{+x\left(-\frac{1}{8}e^{-\frac{1}{8}t}\right)} = 2s_0 e^{-2t} \cancel{-\frac{1}{8}\left(xe^{-\frac{1}{8}t}\right)}$ $\frac{dx}{dt} = 2s_0 e^{-2t}\, e^{\frac{1}{8}t}$ $= 2s_0 e^{-\frac{15}{8}t}$ Integrating this DE directly w.r.t. time t on both sides, $x = \int 2s_0 e^{-\frac{15}{8}t}\, dt$ $= -\frac{16}{15}s_0 e^{-\frac{15}{8}t} + A'$, where A' is an arbitrary constant. Since $x = ce^{\frac{1}{8}t}$, $ce^{\frac{1}{8}t} = -\frac{16}{15}s_0 e^{-\frac{15}{8}t} + A'$ $c = \left(-\frac{16}{15}s_0 e^{-\frac{15}{8}t} + A'\right)e^{-\frac{1}{8}t}$ $= -\frac{16}{15}s_0 e^{-2t} + A'e^{-\frac{1}{8}t}$	Many candidates were unable to fully carry out the substitution procedure to rewrite the DE into one that involves just the variables x, t and dx/dt, some stopping short of substituting away the dc/dt in the DE, as it wasn't found in the first place. Some others attempted to find dc/dt from the given substitution relation, but made mistakes in performing the differentiation, with errors stemming from: - erroneously treating x as a constant, e.g. $\frac{dc}{dt} = x\left(-\frac{1}{8}e^{-\frac{1}{8}t}\right)$ ✘, - incorrect use of the product rule of differentiation, or incorrect use of the chain-rule of differentiation. $\frac{dc}{dt} = e^{-\frac{1}{8}t} + x\left(-\frac{1}{8}e^{-\frac{1}{8}t}\right)$ ✘, miss $\frac{dx}{dt}$. Amongst candidates who could properly rewrite the DE via substitution as required (approx. 35-45%) however, the majority could subsequently solve the DE to find x via the method of direct integration, with a few erroneously omitted the arbitrary constant of integration, and some others chose to stop after rewriting the DE. Credit that were typically secured in answering this part of the question were mainly the method marks.

		At initial time $t=0$, $c=0$ (initial amount of caffeine in the bloodstream) $0=-\frac{16}{15}s_0\,\mathrm{e}^{-2(0)}+A'\mathrm{e}^{-\frac{1}{8}(0)}$, $0=-\frac{16}{15}s_0+A'$ $A'=\frac{16}{15}s_0$ $\therefore c=-\frac{16}{15}s_0\,\mathrm{e}^{-2t}+\frac{16}{15}s_0\,\mathrm{e}^{-\frac{1}{8}t}$ $=-\frac{16}{15}s_0\left(\mathrm{e}^{-2t}-\mathrm{e}^{-\frac{1}{8}t}\right)$ (shown).	
	Point out and explain a model's assumption(s), with reference to given context.	**(iv)** Point out any one of the following simplifying assumptions for the model used: 1. **All** the caffeine that enters the digestive tract is absorbed into the bloodstream, before being eliminated subsequently. 2. Caffeine is consumed **only** at a single point in time (i.e. the initial time $t=0$) and at no other point of time nor over any other time interval. 3. The constant of proportionality for the absorption rate remains at 2 **throughout**. 4. Caffeine from the digestive tract goes **only** into the bloodstream, and is not directly lost from the digestive tract nor lost elsewhere. 5. The rate at which caffeine is absorbed into the bloodstream is **solely dependent** on the amount of caffeine in the digestive tract and not on other factors such as the concentration of caffeine in the	Only about a-third of the candidates overall secured the credit in answering this question part. A significant portion (more than a-third overall) left this part un-attempted. Amongst responses provided, a significant reason for failing to secure credit was due to asserting assumption(s) that were inconsistent with the model used (e.g. "rate of absorption/elimination is assumed to be constant"× – while the rate that's dependent on the respective amounts of caffeine in the digestive tract / bloodstream has to vary with time). Vague / unclear description in the answer, and not answering the question on providing an assumption (e.g. putting down details concerning suitability of model or details concerning a prediction by the model) were other significant reasons for not securing the credit in this part.

		digestive tract / volume of fluid in which caffeine is dissolved / amount of energy available to perform absorption or transportation of caffeine, etc. 6. There is **no maximum limit** to the amount of caffeine that could be present in the digestive tract / bloodstream. 7. The absorption of caffeine from the digestive tract into the bloodstream is **only** a one-way process (no caffeine goes from the bloodstream back into the digestive tract).	

6	Complex Numbers		
	Assessment Objectives	**Solution**	**Feedback**
	Complex roots of polynomial equations	(i) Denote the real root by x, then equating the real and imaginary parts separately, we have $x^3+2x^2+kx+8=0$ $-8\sqrt{2}x-4\sqrt{2}k=0$ So $k=-2x$. Substituting $k=-2x$ into the first equation, we get $x^3+2x^2-2x^2+8=0$ $x^3+8=0$ Since x is real, $x=-2$. Thus, $k=-2x=4$. (ii) The original equation factorizes into $(z+2)\left(z^2+\left(4-8\sqrt{2}\mathrm{i}\right)\right)=0$ Solving $z^2+4-8\sqrt{2}\mathrm{i}=0$, let $z=a+\mathrm{i}b$, where $a,b\in\mathbb{R}$, then $(a+\mathrm{i}b)^2+4-8\sqrt{2}\mathrm{i}=0$ We have $a^2-b^2+4=0$ $2ab-8\sqrt{2}=0$ Then $ab=4\sqrt{2}$ $b=\dfrac{4\sqrt{2}}{a}$ Substituting $b=\dfrac{4\sqrt{2}}{a}$ into $a^2-b^2+4=0$, we get $a^4+4a^2-32=0$	The question was very poorly done in general. In the first part, most candidates managed to find the value of k by substituting the value of $z=-2$ into the equation. However, only a handful candidates considered the separation of the real and imaginary parts based on the given condition that there is a real root. In the second part, many candidates used a very cumbersome algebra to find the factorization of the equation, and half of them even failed to factorize correctly. Again, only a minority of candidates proceeded with denoting the unknown z in cartesian form, as hinted by the question, to solve for the other roots. Quite a number of students took the square root of a complex number straightaway as if they were working with real roots. The answers from the candidates reflect the following: 1. poor understanding of the question when the phrasing varies. In particular, once the value of k is found, which majority did, the second part resembles problems they have encountered in both lectures and tutorials. Yet most were clueless. 2. poor algebraic skills when the presence of letters and surds doubled the challenge of abstractness. There were cases when students failed to solve correctly after getting expressions like "2k=8" or "2=a+2". 3. poor grasp of some basic ideas such as the distinction and relation between "root" and "factor", or the necessity of considering the real and imaginary parts separately when dealing with the algebra of complex numbers.

		$a^2 = 4$ or -8 Since $a^2 \geq 0$, $a^2 = 4$ $a = \pm 2$ $b = \pm 2\sqrt{2}$ The other two roots are $-2-2\sqrt{2}\mathrm{i}$ and $2+2\sqrt{2}\mathrm{i}$	

7	Vectors			
	Assessment Objectives	**Solution**		**Feedback**
	Students to be able to find the Cartesian equation of a plane from a given 3D figure.	**(i)**	The surface is perpendicular to both $\overrightarrow{CD}$ and $\overrightarrow{CB}$, the normal vector is parallel to $\begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix} \times \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$. So we have $\mathbf{n} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$ Scalar product form is given by $\mathbf{r} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 60$ So Cartesian equation is $4x + z = 60$	Most of the students who revised vectors could identify the two vectors parallel to the plane. A number of students did not know that to find the normal vector, it is enough to just find the vector product of the vectors **in the lowest term**, with big value constants appearing at subsequent steps, leading to further mistakes. Several students still are not familiar what Cartesian equation of a plane means, showing that fundamental concepts are not revised properly.
		(ii)	$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OF}\right) = \frac{1}{2}\begin{pmatrix} 3 \\ 34 \\ 24 \end{pmatrix}$ Shortest distance is given by $\left\lvert \frac{\overrightarrow{OB} \times \overrightarrow{OM}}{\left\lvert \overrightarrow{OM} \right\rvert} \right\rvert = \frac{\frac{5}{2}\left\lvert \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ 34 \\ 24 \end{pmatrix} \right\rvert}{\frac{\sqrt{1741}}{2}} = \frac{5\left\lvert \begin{pmatrix} 96 \\ -72 \\ 90 \end{pmatrix} \right\rvert}{\sqrt{1741}} = \frac{750}{\sqrt{1741}}$	A common mistake is to assume that $\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{FA}\right)$. The correct method is to apply Ratio Theorem. Many students knew that they need to apply concept of length of projection but used the wrong pair of vectors. It is important to draw a diagram to check! Quite a number of students also did not express the answer in exact form, despite the question clearly stating so.

			Alternatively, use $\left\lvert \frac{\overrightarrow{OB}\cdot\overrightarrow{OM}}{\lvert\overrightarrow{OM}\rvert} \right\rvert = \frac{\frac{5}{2}\left\lvert\begin{pmatrix}3\\4\\0\end{pmatrix}\cdot\begin{pmatrix}3\\34\\24\end{pmatrix}\right\rvert}{\frac{\sqrt{1741}}{2}} = \frac{725}{\sqrt{1741}}$ Followed by Pythagoras Theorem, shortest distance is $\sqrt{OB^2 - \frac{725^2}{1741}} = \frac{750}{\sqrt{1741}}$	There are a number of wrong or sloppy use of notations.
	Students to be able to find the point of intersection of two lines, in context of a 3D figure.	**(iii)**	OG has equation: $\mathbf{r} = \lambda\begin{pmatrix}1\\2\\8\end{pmatrix}$ while AF has equation: $\mathbf{r} = \begin{pmatrix}0\\20\\0\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\8\end{pmatrix}$, **where** $\lambda,\mu \in \mathbb{R}$ At point of intersection, $\lambda = \mu$ **--------(1)** $2\lambda = 20 - 2\mu$ **-------- (2)** Solving, we have $\lambda = \mu = 5$	Although most students could write down the equation of a line, the equations are either not complete or with the **"r"** missing at the start of the equation.
	Students to be able to find angle between two planes in context.	**(iv)**	$\cos\theta = \frac{\begin{pmatrix}4\\0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{17}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{17}}\right) = 1.33rad$ $or\ 76.0^\circ$	Most students could do this part. However, some students rely on the given figure to identify the required angle which is not necessary. They need to remember that angle between two planes is simply the angle between their normal vectors.

Section B: Probability and Statistics [34 marks]

8	Permutation and Combination, Probability		
	Assessment Objectives	**Solution**	**Feedback**
	Ability to find the number of arrangements of objects in a circle	**(i)** Number of different seating arrangement $=(12-1)!=39916800$	Majority of the students who attempted this part of the question could get the answer. Common mistakes made include: 1. Fail to read question properly and did not consider the circular arrangement. 2. $\frac{(12-1)!}{3!4!5!}$ treating the students from the various schools as identical.
	Ability to find the number of arrangements of objects in a circle with restrictions (separation)	**(ii)** Number of arrangement where all students from College of Arts are separated $=(9-1)!\ \times {}^9C_3 \times 3!$ $=20321280$ $\mathrm{P}(A)$ $=\frac{20321280}{39916800}=\frac{28}{55}$	Majority of the students were weak in calculating the number of arrangement where all students from College of Arts are separated. Students who attempted the complement method only considered the case where all 3 were seated together and fail to realise the case where 2 students may sit together. In addition, for students who realized the case where 2 students could sit together, they gave the answer $10!\ \times 2!$ without realizing this value is double counted.
	Ability to calculate conditional probabilities involving arrangements of objects in a circle with restrictions (group and separation)	**(iii)** $\mathrm{P}(A\cap B)$ $=\frac{(6-1)!\ \times 4!\ \times {}^6C_3 \times 3!}{39916800}=\frac{345600}{39916800}=\frac{2}{231}$ $\mathrm{P}(B\mid A)$ $=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}$ $=\frac{2/231}{28/55}=\frac{5}{294}$	Majority of the students could recall the conditional probability formula but fail to calculate the respective probabilities required for the computation. Common mistakes made include: 1. $\mathrm{P}(B\mid A)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ 2. $\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B)$ 3. $\mathrm{P}(A\cap B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cup B)$ but could not find $\mathrm{P}(A\cup B)$.

	Ability to determine independent events	**(iv)** $\mathrm{P}(B)=\dfrac{(9-1)!\ \times 4!}{39916800}=\dfrac{967680}{39916800}=\dfrac{4}{165}$ From part (iii), $\mathrm{P}(B\mid A)=\dfrac{5}{294}$. Since $\mathrm{P}(B\mid A)\neq \mathrm{P}(B)$, A and B are not independent.	Most students were able to recall the condition for independence but were weak in the algebraic manipulation which resulted in a loss of rk. Common mistakes made include 1. Attempting to show $\mathrm{P}(B\mid A)=\mathrm{P}(A)$. 2. Attempting to show $\mathrm{P}(B\mid A)\neq 0$. 3. Attempting to show $\mathrm{P}(A)\mathrm{P}(B)\neq 0$.
	Ability to find maximum value of a parameter given the range of a probability HOT	**(v)** $\mathrm{P}(\text{all prizes are given to students from College of Science})=\dfrac{{}^{5}C_{p}}{{}^{12}C_{p}}$ Using GC, $\begin{array}{c\|ll} p & \dfrac{{}^{5}C_{p}}{{}^{12}C_{p}} & \\ \hline 3 & 0.04545 & >0.01 \\ 4 & 0.0101 & >0.01 \\ 5 & 0.00126 & <0.01 \end{array}$ Hence, maximum p is 4.	Almost all students could not do this part of the question and left it blank. Only 3 students in the cohort could do this part of the question although the previous parts were not well attempted.

9 Normal and Sampling Distribution

Assessment Objectives	Solution	Feedback
Able to solve for μ and σ by sketching normal curve and performing standardization.	**(i)** $\mathrm{P}(X>9)=1-\mathrm{P}(X<9)$ $=1-0.97725$ $=0.02275$ Since $\mathrm{P}(X<7)=\mathrm{P}(X>9)=0.02275$, $\mu=\frac{7+9}{2}=8$ $\mathrm{P}(X<7)=0.02275$ $\mathrm{P}\left(Z<\frac{7-8}{\sigma}\right)=0.02275$ Using G.C. $\frac{7-8}{\sigma}=-2.00000445$ $\sigma=0.5$	Majority of students scored full marks for this part. However, only a handful of them solved for μ by symmetry. Instead, they standardized twice and solved by simultaneous equations. Students who made mistakes confused the continuous random variable with discrete. Mistakes include • table method to show σ • rewrite $\mathrm{P}(X<7)$ as $\mathrm{P}(X\le 6)$ before standardization. Other mistakes include erroneously writing standardization formula as $\frac{\bar{x}-\mu}{\sigma^2}$
Able to find distribution of sum and multiples of independent, normal random variables.	**(ii)** $\mathrm{P}(X>6)\times\mathrm{P}(Y>6)$ $=0.999968314\times0.0148150053$ $=0.0148$	A large number of students made mistakes in this part. They include • rewrite $\mathrm{P}(X>6)$ as $1-\mathrm{P}(X\le 5)$ • add $\mathrm{P}(X>6)$ and $\mathrm{P}(Y>6)$ • leave $\mathrm{P}(X>6)$ and $\mathrm{P}(Y>6)$ as 2 separate answers. These students misinterpreted the requirement of the question.
	(iii) $X+Y\sim\mathrm{N}\left(8+4.26,0.5^2+0.8^2\right)$ $\sim\mathrm{N}(12.26,0.89)$ $\mathrm{P}(X+Y>12)$ $=0.609$	Majority of students scored full marks (or 1 mark) for this part. Mistakes include • add standard deviation instead of variance • key in σ^2 instead of σ in the G.C.
	(iv) There are less cases in (ii) compared to (iii). For example, the case where $x=5$ and $y=8$ is belongs in (iii) but not in (ii).	This part was poorly attempted. Mistakes include • Describe air pressures in basketballs and volleyballs. These answers were irrelevant.

			• (ii) is a subset of (iii), without further explanation/examples. These answers were deemed insufficient.
		(v) $Y_1+Y_2+...+Y_6-3X \sim N\left(6(4.26)-3(8), 6\left(0.8^2\right)+3^2\left(0.5^2\right)\right)$ $\sim N(1.56, 6.09)$ $P\left(Y_1+Y_2+...+Y_6<3X\right)$ $=P\left(Y_1+Y_2+...+Y_6-3X<0\right)$ $=0.264$	A large number of students made mistakes in this part. They include • consider $6Y$ instead of $Y_1+Y_2+...+Y_6$ • subtract instead of add variance • mix up X and Y
	Able to state condition for sum and multiples of independent, normal random variables.	**(vi)** Air pressure in basketballs and volleyballs are independent.	This part was poorly attempted. Many students were unable to specify independence between the correct variables. Mistakes include • probabilities were independent • choice of basketball and volleyballs were independent • all events were independent Other mistakes include assuming • μ and σ are fixed • air pressure is not affected by external factors These assumptions were not accepted because they are not specific to assumptions needed for calculations of random variables undergoing linear transformations.

10. Discrete Random Variable & Binomial Distribution

<table>
<tr><th>Assessment Objectives</th><th>Solution</th><th>Feedback</th></tr>
<tr><td>Able to apply concept that sum of probabilities in a probability distribution table is 1.</td><td>(i)
$\frac{1}{6}+\frac{1}{3}+a+\frac{1}{4}+\frac{1}{12}=1$
$a=1-\frac{5}{6}=\frac{1}{6}$</td><td>Most could get this part correct.</td></tr>
<tr><td>Able to find E(X) and Var(X) from a probability distribution table of a discrete random variable.</td><td>(ii)
<table><tr><td>Grade</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr><tr><td>Price $$X$</td><td>10</td><td>13</td><td>16</td><td>19</td><td>22</td></tr><tr><td>$P(X=x)$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{12}$</td></tr></table>Expected revenue,
$E(X)=10\times\frac{1}{6}+13\times\frac{1}{3}+16\times\frac{1}{6}+19\times\frac{1}{4}+22\times\frac{1}{12}$
$=\$15.25$ (exact value)</td><td>Most could get this part right.
A few made copying mistakes when calculating E(X) thus leading to error in Var(X) as well.</td></tr>
<tr><td></td><td>$\text{Var}(X)=E(X^2)-[E(X)]^2$
$=\frac{10^2}{6}+\frac{13^2}{3}+\frac{16^2}{6}+\frac{19^2}{4}+\frac{22^2}{12}-[15.25^2]$
$=\frac{219}{16}$
$=13.6875$ (exact value)</td><td>Most students could get this part correct. A few could not remember the formula for Var(X).</td></tr>
<tr><td>Able to find the most likely value of a binomial distribution (as distinct from finding the mean of a binomial distribution)</td><td>(iiia) Let Y be the random variable denoting the number of bottles of detergent with a pink towel out of 19 bottles.
$X\sim B(19,0.2)$
Using GC:
<table><tr><th>y</th><th>$P(Y=y)$</th></tr><tr><td>2</td><td>0.1540</td></tr><tr><td>3</td><td>0.2182</td></tr><tr><td>4</td><td>0.2182</td></tr><tr><td>5</td><td>0.1636</td></tr></table></td><td>About half the students used the GC and obsvd the probabilities to 4 d.p. and hence gave mode as 3 and 4.

Many students also confused mean with mode and used $np=19\times0.2=3.8\approx4$ as the mode. Hopefully, making an error here will allow students to note the correct way to obtain the mode.

Some issues:
However, many students using the latest model GC TI-84 Plus CE moved their cursor to the Y_1 column and observed two different values for</td></tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hence, the most likely number of bottles is 3 and 4.</td>
<td>$\mathrm{P}(X = 3) = 0.21819940194575$ and
$\mathrm{P}(X = 4) = 0.2181994019463$
i.e. difference in the 12th and 13th d.p. and hence gave the mode only as 4 which is incorrect. In the older versions of GC, the table values are only up to the 11th d.p. and hence there is no difference in the probabilities. This is due to the limitation of the GC.

Using the binomial distribution formula,
$\frac{\mathrm{P}(X = 3)}{\mathrm{P}(X = 4)} = \frac{{}^{19}C_3(0.2)^3(0.8)^{16}}{{}^{19}C_4(0.2)^4(0.8)^{15}} = 1$
Hence the mode is 3 and 4.</td>
</tr>
<tr>
<td></td>
<td>Formulate an inequality and use GC to solve for sample size N of a binomial distribution.</td>
<td>(iiib)</td>
<td>Let W be the random variable denoting the number of bottles of detergent with a pink towel out of N bottles.
$W \sim \mathrm{B}(N, 0.2)$
$\mathrm{P}(W \geq 2) > 0.5$
$1 - \mathrm{P}(W \leq 1) > 0.5$
Using GC:
<table>
<tr><td>w</td><td>$1 - \mathrm{P}(W \leq 1)$</td></tr>
<tr><td>8</td><td>0.4967</td></tr>
<tr><td>9</td><td>0.5638</td></tr>
<tr><td>10</td><td>0.6242</td></tr>
</table>

Hence least value of N is 9.</td>
<td>Students who studied this topic could get the correct. Common errors include not expressing the inequality correctly and hence simplifying the expression incorrectly and drawing the wrong conclusion from the GC table, and expressing final answer as $N = 9$ when in fact N is a range of values with 9 being the least value.</td>
</tr>
<tr>
<td></td>
<td>Solving for p using the binomial formula
HOT</td>
<td>(iv)</td>
<td>Let V be the random variable denoting the number of bottles of detergent with a pink towel out of 20 bottles.
Then $V \sim \mathrm{B}(20, p)$
Given $\mathrm{P}(V = 10) = 0.003237$
${}^{20}C_{10}p^{10}(1-p)^{10} = 0.003237$
$p(1-p) = \left(\frac{0.003237}{184756}\right)^{\frac{1}{10}}$</td>
<td>Many could get the formula from MF26 and express the equation correctly based on the question. However, many also did not know how to use the GC to obtain the final answer.

The main problem for not being able to set up the equation correctly is not understanding the situation in the question or not learning the binomial formula correctly.</td>
</tr>
</table>

		$p^2 - p + 0.1676310305 = 0$ Hence, $p = 0.2130000531$ or 0.7869999469 $p \approx 0.213$ or 0.787	