Outline of discussion points

- Repeating/periodic functions $f(x+a) = f(x)$ or $f(x) = f(x-b)$ (periods are a, b respectively).
  - Basic concepts from transformations
  - Manipulation of numbers
- Piecewise functions
  - Basic concepts
  - Use of GC
  - Be careful of end-points
- $\triangleright$ School notes example 17: combination of the 2
- $\triangleright$ Tutorial question Q9 (see below)
- Extra: advanced function manipulation
  - Discussion based on ideas from basic algebra and trigo
  - Advanced way of thinking of logarithms and exponentials
  - $ff^{-1}(x)$ and $f^{-1}f(x)$.
- $\triangleright$ Tutorial question Q7(ii), 8(ii) (see below)
- $\triangleright$ TYS Questions using these ideas: Pg 1 Q7 (2012), Pg 2 Q5(i) (2013), Pg 3 Q1 (2013)
- Extra: "self-inverse" functions
- Odd and even functions:
  - Even function: $f(x) = f(-x)$. Symmetrical about the y-axis.
  - Odd function $f(-x) = -f(x)$. Symmetrical about the origin.
  - Uses: algebra and roots/x-intercepts. In integration and area.

Questions from school tutorial

Q9

Sketching of Q9 can be done based on our discussion on piecewise functions. To find f^{-1} for piecewise functions, we repeat our steps for finding inverse (the rule as well as the domain) for the individual pieces.

Q7(ii)

For Q7(ii), the longer, systematic way to approach will be to continue working with composite functions.

$$f^3(x) = fff(x) = f(f^2(x)) = f(\frac{x-1}{x}) = \frac{1}{1 - \frac{x-1}{x}} = \dots$$

After quite a bit of algebraic simplification we can get the answer $f^3(x) = x$. But turns out there's a faster method! We can make use of part (i).

$$f^3(x) = ff^2(x) = ff^{-1}(x) = x.$$

Q8(ii)

For Q8(ii), there's not much we can do except play around with algebra and substitution. We have $hf(x) = 4x^2 - 4x - 7$ which means that

$$h(2x-1) = 4x^2 - 4x - 7.$$

Our aim is to find $h(x)$. How can we get rid of the 2? How about the -1?

Extension of Q8(ii)

A modification of Q8(ii) will involve the question telling us that

$$fh(x) = 4x^2 - 4x - 7$$

and ask us to find $h(x)$. We can do a substitution approach as well, but there's an easier way.