5 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ such that

$$\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{b} = \mathbf{i} + 2\mathbf{j}.$$

The point C has position vector $\mathbf{c}$ given by $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$, where λ and μ are positive constants.

(i) Given that the area of triangle OAC is $\sqrt{(126)}$, find μ. [4]

(ii) Given instead that $\mu = 4$ and that $OC = 5\sqrt{3}$, find the possible coordinates of C. [4]

6 **Do not use a calculator in answering this question.**

The complex number z is given by $z = 1 + \mathrm{i}c$, where c is a non-zero real number.

(i) Find z^3 in the form $x + \mathrm{i}y$. [2]

(ii) Given that z^3 is real, find the possible values of z. [2]

(iii) For the value of z found in part **(ii)** for which $c < 0$, find the smallest positive integer n such that $|z^n| > 1000$. State the modulus and argument of z^n when n takes this value. [4]

7 A function f is said to be self-inverse if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

The function g is defined by

$$\mathrm{g} : x \mapsto \frac{x+k}{x-1}, \quad x \in \mathbb{R}, \ x \neq 1,$$

where k is a constant, $k \neq -1$.

(i) Show that g is self-inverse. [2]

(ii) Given that $k > 0$, sketch the curve $y = \mathrm{g}(x)$, stating the equations of any asymptotes and the coordinates of any points where the curve crosses the x- and y-axes. [3]

(iii) State the equation of one line of symmetry of the curve in part **(ii)**, and describe fully a sequence of transformations which would transform the curve $y = \dfrac{1}{x}$ onto this curve. [4]

8 The curve C has equation

$$x - y = (x + y)^2.$$

It is given that C has only one turning point.

(i) Show that $1 + \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{2x + 2y + 1}$. [4]

(ii) Hence, or otherwise, show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\left(1 + \dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3$. [3]

(iii) Hence state, with a reason, whether the turning point is a maximum or a minimum. [2]

1 Planes p, q and r have equations $x - 2z = 4$, $2x - 2y + z = 6$ and $5x - 4y + \mu z = -9$ respectively, where μ is a constant.

(i) Given that $\mu = 3$, find the coordinates of the point of intersection of p, q and r. [2]

(ii) Given instead that $\mu = 0$, describe the relationship between p, q and r. [3]

2 It is given that

$$y = \frac{x^2 + x + 1}{x - 1}, \quad x \in \mathbb{R},\ x \neq 1.$$

Without using a calculator, find the set of values that y can take. [5]

3 **(i)** Sketch the curve with equation

$$y = \frac{x + 1}{2x - 1},$$

stating the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. [4]

(ii) Solve the inequality

$$\frac{x + 1}{2x - 1} < 1.$$ [2]

4 The complex number w is given by $1 + 2\mathrm{i}$.

(i) Find w^3 in the form $x + \mathrm{i}y$, showing your working. [2]

(ii) Given that w is a root of the equation $az^3 + 5z^2 + 17z + b = 0$, find the values of the real numbers a and b. [3]

(iii) Using these values of a and b, find all the roots of this equation in exact form. [3]

5 It is given that

$$\mathrm{f}(x) = \begin{cases} \sqrt{\left(1 - \dfrac{x^2}{a^2}\right)} & \text{for } -a \leqslant x \leqslant a, \\ 0 & \text{for } a < x < 2a, \end{cases}$$

and that $\mathrm{f}(x + 3a) = \mathrm{f}(x)$ for all real values of x, where a is a real constant.

(i) Sketch the graph of $y = \mathrm{f}(x)$ for $-4a \leqslant x \leqslant 6a$. [3]

(ii) Use the substitution $x = a\sin\theta$ to find the exact value of $\displaystyle\int_{\frac{1}{2}a}^{\frac{\sqrt{3}}{2}a} \mathrm{f}(x)\,\mathrm{d}x$ in terms of a and π. [5]

Section A: Pure Mathematics [40 marks]

1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{2+x}{1-x}, \quad x \in \mathbb{R}, \quad x \neq 1,$$

$$\mathrm{g} : x \mapsto 1 - 2x, \quad x \in \mathbb{R}.$$

(i) Explain why the composite function fg does not exist. [2]

(ii) Find an expression for $\mathrm{gf}(x)$ and hence, or otherwise, find $(\mathrm{gf})^{-1}(5)$. [4]

2

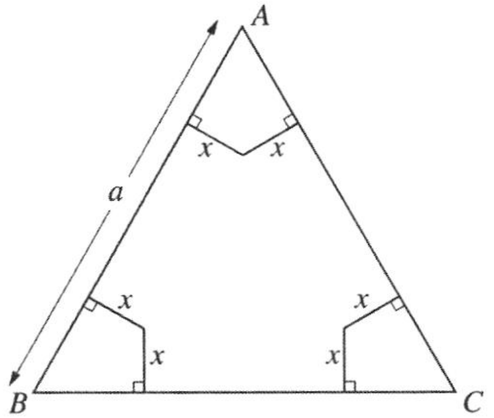

Fig. 1

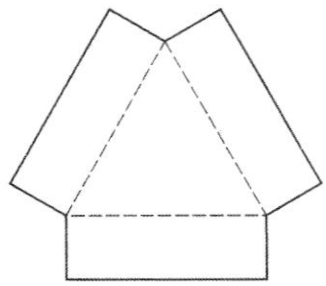

Fig. 2

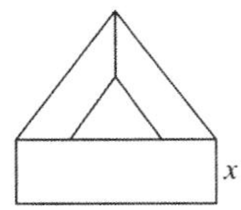

Fig. 3

Fig. 1 shows a piece of card, ABC, in the form of an equilateral triangle of side a. A kite shape is cut from each corner, to give the shape shown in Fig. 2. The remaining card shown in Fig. 2 is folded along the dotted lines, to form the open triangular prism of height x shown in Fig. 3.

(i) Show that the volume V of the prism is given by $V = \frac{1}{4}x\sqrt{3}(a - 2x\sqrt{3})^2$. [3]

(ii) Use differentiation to find, in terms of a, the maximum value of V, proving that it is a maximum. [6]

3 **(i)** Given that $\mathrm{f}(x) = \ln(1 + 2\sin x)$, find $\mathrm{f}(0)$, $\mathrm{f}'(0)$, $\mathrm{f}''(0)$ and $\mathrm{f}'''(0)$. Hence write down the first three non-zero terms in the Maclaurin series for $\mathrm{f}(x)$. [7]

(ii) The first two non-zero terms in the Maclaurin series for $\mathrm{f}(x)$ are equal to the first two non-zero terms in the series expansion of $\mathrm{e}^{ax}\sin nx$. Using appropriate expansions from the List of Formulae (MF15), find the constants a and n. Hence find the third non-zero term of the series expansion of $\mathrm{e}^{ax}\sin nx$ for these values of a and n. [5]