Permutations and Combinations

October 9, 2017

1. **[ACJC Prelims 17]**
   A school is asked to send a delegation of 7 students to attend the opening ceremony of the Asian Youth Games. They are chosen from 8 swimmers, 5 basketball players and 5 tennis players where no student plays more than one game.

   How many different delegations can be formed? [1]

   One of the swimmers is the brother of a basketball player. How many different delegations can be formed which include exactly one of the two brothers? [2]

   Find the probability that the delegation consists of at least 2 students from each sport. [3]

2. **[AJC Prelims 17]**
   A salad bar in a restaurant has 7 types of greens, 3 types of proteins and 6 types of toppings. There are also 2 types of soup and 2 types of yogurt for selection.

   A promotional set meal consists of a salad plate, plus either a soup or a yoghurt. For the salad plate, a customer needs to choose 3 different types of greens, 1 type of protein, and 2 different types of toppings.

   (a) Find the number of ways the customers may customise his set meal. [2]

   Each morning, the employee has to key a password to access the company accounts. The password consists of 3 digits from 1 to 9, followed by 2 letters of the alphabet. Each digit or letter may be used any number of times.

   (b) there is no restriction, [1]

   (c) the password has exactly one even digit and at least one vowel. [3]

   One morning, the employee forgot the password. However, he is certain that the digits are all different, but the alphabets are identical. He makes an attempt to type in the password.

   (d) Find the probability that the employee gets the password correct in his first attempt. [2]

3. **[DHS Prelims 17]**
The Tan family has 2 children while the Wong family has 3 children. The children, together with both their parents, catch a movie at the cinema. At the ticket counter, they realise only 2 rows of consecutive empty seats are left, where Row L has 5 seats and Row M has 4 seats.

Find the number of different possible arrangements if

(a) there are no restrictions, [1]

(b) the Tan siblings must not sit together. [3]

The 9 or them are randomly seated. Find the probability that the Tan siblings sit between their parents given that the Wong family takes Row L. [2]

4. **[HCI Prelims 17]**
In an IT department, a staff is tasked to form 7-letter codes (need not be valid words) using the given word 'SPECIAL'. Find the number of codes that can be formed if

(a) there are no restrictions except the code 'SPECIAL' cannot be formed, [2]

(b) all the 3 vowels cannot be together,

(c) the first and last letters are consonants. [2]

5. **[RI Prelims 17]**
Find the number of ways in which the letters of the word SECTION can be arranged if

(a) the letters are not in alphabetical order, [1]

(b) the consonants (S,C,T,N) and vowels (E,I,O) must alternate, [2]

(c) all the vowels are together, [2]

(d) all the vowels are separated, [2]

(e) there must be exactly two letters between the two letters E and O. [2]

Find, as a fraction in its lowest term, the probability that after arranging the letters of the word SECTION, there is at least one consonant and at least one vowel between the two letters E and O. [4]

6. **[RVHS Prelims 17]**
The school ICT assistant needs to schedule a group of 4 Science, 3 Mathematics and 2 Humanities Department teachers for upgrade of their laptops. He decides to arrange the 9 teachers in random order. On a particular day, find the probability that

   (a) Miss Tan from the Science Department is first and Mr Ng from the Humanities Department is the last teacher to have their laptops upgraded, [2]

   (b) teachers from the same department are randomly arranged to have their laptops upgraded before teachers from another department to have their laptops upgraded in random order, [2]

   (c) given that the 5 teachers have their laptops upgraded by noon time, there are exactly 2 from the Science Department and exactly 2 from the Mathematics Department. [2]

7. **[SAJC Prelims 17]**
Thomas has six tiles, each with a different letter of his name on it. Thomas randomly arranges these letters in a line. Find the probability that the six tiles are arranged

   (a) in the correct order that spells his name. [2]

   (b) such that the vowels are separated. [2]

   (c) such that the vowels are at the two ends. [2]

8. **[SRJC Prelims 17]**
Find how many different arrangements can be made using all letters of the word PRELIMS if

   (a) there are no restrictions, [1]

   (b) the first and last letters must both be vowels, [2]

   (c) the letters R,L, and M must be together, [2]

   (d) the letters R,L, and M must be separated. [3]

9. **[TJC Prelims 17]**
Find the number of 6-letter passwords that can be formed using the letters from the word SINGAPORE if

   (a) repetitions of letters are not allowed. [1]

   (b) at least two vowels must be chosen and repetition of letters are not allowed. [2]

   (c) three distinct vowels and three distinct consonants are chosen, and vowels and consonants must alternate? [3]

10. **[YJC Prelims 17]**
A chess team of 5 players is to be selected from 15 boys. In how many ways can the team be chosen if

   (a) no more than one of the three best players is to be included, [2]

   (b) at least one of the 4 youngest players is to be included? [2]

Answers

1. 31824.
   16016.
   0.352.

2. (a) 6300.
   (b) 492804.
   (c) 70500.
   (d) $\frac{1}{13104}$.

3. (a) 362880.
   (b) 292 329.
   (c) $\frac{1}{6}$.

4. (a) 5049.
   (b) 4320.
   (c) 1440.

5. (a) 5039.
   (b) 144.
   (c) 720.
   (d) 1440.
   (e) 480.

6. (a) $\frac{1}{72}$.
   (b) $\frac{1}{210}$.
   (c) $\frac{2}{7}$.

7. (a) $\frac{1}{720}$.
   (b) $\frac{2}{3}$.
   (c) $\frac{1}{15}$.

8. (a) 5040.
   (b) 240.
   (c) 720.
   (d) 1440.

9. (a) 60 480.
   (b) 57 600.
   (c) 2880.

10. (a) 2277.
    (b) 2541.

Sampling

1. **[SRJC Prelims 17]**

   (a) The manager of a bookstore wishes to conduct a survey to seek the customers opinions on its opening hours. If the manager decides to survey a sample of the first 80 customers who leave the bookstore, give a reason why this sample may not be appropriate. [1]

   (b) A surveyor decides to obtain a random sample of 20 residents from the apartment block. He randomly selects 20 units from the apartment block and chooses one resident from each unit.

      i. In the context of the question, explain what is meant by the term 'random sample'. [1]

      ii. Explain why this method may not be appropriate. [1]

      iii. Describe an alternative method so that the surveyor will choose a sample of 20 residents at random from the apartment block of 100 residents. [1]

Answers

1. (a) The first 80 customers may not be representative of all bookstore customers. The mid-day and late night shoppers will be unrepresented.

   (b) i. In the context of the question, the term random sample means that every resident of the same apartment block has the same probability of being selected. The selection of residents is independent.

      ii. Since each unit has different number of people, so the probability of being chosen is not equally likely.

      iii. Obtain the name list of the 100 residents living at the apartment block and assign a number from 1 to 100 to all the residents. Use a computer program to generate 20 random numbers from 1 to 100. The person who is assigned the chosen number will be selected.