MERIDIAN JUNIOR COLLEGE
JC2 Mid Year Examination
Higher 2

H2 Mathematics 9758/01

Paper 1 **25 June 2018**

3 Hours

Additional Materials: Writing paper
Graph Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 8 printed pages.

[Turn Over

Section A: Pure Mathematics [60 marks]

1 Solve the simultaneous equations

$$3z-4w=6+\mathrm{i}, \qquad 2w^2-6z+2w=-18-4\mathrm{i},$$

giving each answer in the form $a+b\mathrm{i}$, where a and b are real. [5]

2 Yellow-faced bees in Hawaii are under threat from habitat loss and could become extinct if conservation efforts are not taken. A recent wildfire caused a drastic decrease in the population of Yellow-faced bees in Hawaii. Before the wildfire, the population of Yellow-faced bees (in hundreds) in Hawaii is k, where k is a constant.

The population of Yellow-faced bees (in hundreds) t months after the start of the wildfire is y. After the start of the wildfire, conservation efforts were made to preserve the population of the Yellow-faced bees. It is found that the Yellow-faced bees' population (in hundreds) can be modelled by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{3t^2+t(y-k)+2(y-k)}{t(t+2)}, \quad t>0.$$

(i) Using the substitution $y-k=xt$, show that the above differential equation can be reduced to

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{3}{t+2}.$$ [3]

It is given that the total number of Yellow-faced bees in Hawaii is back to k one year after the wildfire.

(ii) Express y in terms of t and k. [5]

(iii) Hence, by considering the graph of y against t or otherwise, deduce the minimum value of the initial population such that the Yellow-faced bees will not become extinct because of the wildfire. [2]

3 (i) Given $u_r = \frac{r^2}{2^r}$, show that $u_r - u_{r+1} = \frac{(r-1)^2}{2^{r+1}} - \frac{1}{2^r}$ [2]

(ii) Hence, find $\sum_{r=1}^{n}\left(\frac{(r-1)^2}{2^{r+1}} - \frac{1}{2^r}\right)$. [3]

(iii) By considering $\sum_{r=1}^{n}\frac{(r-1)^2}{2^{r+1}}$ or otherwise, show that $\sum_{r=1}^{n}\frac{r^2}{2^r} = 6 - \frac{(n+2)^2}{2^n} - \frac{1}{2^{n-1}}$

[5]

4 Let $y = \frac{\sin 2x}{1+\sin^2 x}$.

(i) Show that $(\sin 2x)y + (1+\sin^2 x)\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2x$. [2]

(ii) By further differentiation of the result in **(i)**, find the Maclaurin's series for y, up to and including the term in x^3. [5]

(iii) Hence find the Maclaurin's series for $\ln(1+\sin^2 x)$, up to and including the term in x^4. [3]

5 A student models the flight trajectory of a badminton shuttlecock after a serve made by a player using the curve C with parametric equations

$$x = 5 - e^p, \quad y = \frac{p}{e^{2p}} \quad \text{for } 0 \le p \le \ln 5.$$

(a) **(i)** Find the exact coordinates of the stationary point of curve C. [5]

(ii) Hence give a sketch of curve C, indicating the coordinates of any stationary point and axial intercepts. [2]

A badminton player intercepts the serve at the point where $p = 1$. The interception would result in the flight trajectory of the shuttlecock moving in a straight line along the tangent to the path at the point of its interception.

(iii) Find an equation of the tangent to the path at the point where $p = 1$, leaving your answer in the form $y = \frac{A}{e^3}x + \frac{B}{e^3} + \frac{C}{e^2}$, where A, B and C are integers to be determined. [3]

(b) **(i)** Show that curve C has Cartesian equation $y = \frac{\ln(5-x)}{(5-x)^2}$. [1]

(ii) The region bounded by curve C and the axes is rotated 2π radians about the x-axis. Find the volume of the solid obtained, giving your answer to 4 significant figures. [2]

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6 **(a)** With respect to the origin O, the position vectors of two points A and B are given by $\mathbf{a}$ and $\mathbf{b}$ respectively where $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$. The angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$. Point C lies on the line segment on AB such that $AC = 2CB$.

(i) Find the position vector of the point C in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Show that the length of projection of OC onto OB is $\frac{3}{2}$ and deduce the position vector of the foot of the perpendicular, N, from C to the line OB. [4]

(b) A point A with position vector $\overrightarrow{OA} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$, where α, β and γ are real constants, has direction cosines $\cos\theta$, $\cos\phi$ and $\cos\delta$, where θ, ϕ and δ are the angles $\overrightarrow{OA}$ makes with the positive x, y and z-axes respectively.

(i) Express the direction cosines $\cos\theta$, $\cos\phi$ and $\cos\delta$ in terms of α, β and γ.

Hence find the value of $\cos^2\theta + \cos^2\phi + \cos^2\delta$. [2]

(ii) Using the result in part **(i)**, find the value of $\cos 2\theta + \cos 2\phi + \cos 2\delta$. [2]

(iii) The vector $\mathbf{d}$ makes angle of $45°$ with the x-axis, $60°$ with the y-axis and δ with the z-axis, where $0° \le \delta \le 90°$. Find the value of δ. If the magnitude of $\mathbf{d}$ is 12, express $\mathbf{d}$ in cartesian form. [3]

Section B: Statistics [40 marks]

7 This question is about arrangements of all nine letters in the word HALLOWEEN.

(i) Find the number of different arrangements of the nine letters that can be made. [2]

(ii) Find the exact probability that the letters can be arranged with exactly one pair of identical letters together. [3]

(iii) Find the exact probability that the letters can be arranged with no two adjacent letters the same. [3]

8 Based on past records, there is a probability of p that a customer with a hotel reservation will not show up. State two assumptions needed such that the number of customers who do not show up at the hotel may be well modelled by a Binomial Distribution. [2]

Assume now that these assumptions do in fact hold.

Hotel TM has a capacity of 200 rooms. In order to maximize profits, Hotel TM will accept more reservations than their capacity. The hotel is overbooked if there are not enough rooms available for customers who show up on that day.

(i) Given that $p = 0.08$, find the probability that Hotel TM is not overbooked when 220 reservations are accepted for the day. [2]

(ii) The probability that Hotel TM is overbooked when 210 reservations are accepted is 0.12. Find the value of p. [3]

9 A bag contains 11 identical balls of which three are red and eight are blue. Three balls are selected at random without replacement from the bag. Let X denote the number of red balls selected out of the three balls.

(i) Show that $P(X=1)=\frac{28}{55}$ and hence write down the probability distribution for X. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [2]

It is further given that one point is awarded for every red ball selected and two points is awarded for every blue ball selected. Also, when there are exactly two balls of the same color selected, one point is deducted from the total number of points.

(iii) Find the probability that exactly two red balls are selected, given that the total number of points is three. [2]

10 Salmon and mackerel are sold by weight. The masses, in kg, of salmon and mackerel are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean Mass	Standard Deviation
Salmon	μ	σ
Mackerel	1.2	0.3

Salmon are sold at \$17 per kg and mackerel are sold at \$15 per kg.

(i) Given that 24% of the salmon were sold for less than \$48.60 and 30% of the salmon were sold for more than \$67.50. Show that μ is 3.5, correct to 1 decimal place and find the value of σ to 1 decimal place. [5]

(ii) Find the probability that the total selling price of a randomly chosen salmon and a randomly chosen mackerel is more than \$95. [3]

11 A gaming company claims that gamers spend an average of m hours daily on a new game that they have launched. A random sample of 30 gamers was surveyed and the time, x hours, spent daily on the game was recorded. The results are summarized by

$$\sum x = 126, \qquad \sum x^2 = 550.1525.$$

(i) Calculate unbiased estimates of the population mean and variance of the number of hours a gamer spends daily on the game. [2]

A test is carried out, at the 5% significance level, to determine whether the average time spent daily on the game has been overstated by the company.

(ii) State appropriate hypotheses for the test. [1]

(iii) Find the range of values of m for which the null hypothesis is not rejected. [3]

A second sample of 50 gamers is surveyed and the time, $\cancel{x}$ y hours, spent daily on the game was recorded, and the results are summarized by

$$\sum y = 215, \qquad \sum y^2 = 964.19.$$

(iv) The two samples are combined into a single sample and a test is carried out at the 5% significance level to determine whether the company's claim is valid. Given that the value of m is 4.5, find the p-value of this test. [4]

End of Paper

2018 H2 MATH (9758/01) JC 2 MYE
SUGGESTED SOLUTIONS WITH MARKER'S COMMENTS

Question 1
Level of Difficulty: Low
General Comments:

- Poorly attempted by majority. Quadratic formula should be used when you have a quadratic equation after eliminating z.
- DO NOT introduce $z = x + yi$ and $w = a + bi$ UNNECESSARILY!! You are introducing more unknowns and because of the w^2 in this question, you end up with very complex equations to solve. You should only adopt the method of letting $z = x + yi$ when you have z and z^* within an equation.

Qn	Solution
1	**Complex Numbers**

$3z - 4w = 6 + \mathrm{i}$ --- (1)
$2w^2 - 6z + 2w = -18 - 4\mathrm{i}$ ---- (2)

(1) x 2: $6z - 8w = 12 + 2\mathrm{i}$ --- (3)

(3)+(2): $2w^2 - 6w = -6 - 2\mathrm{i}$

$$2w^2 - 6w + 6 + 2\mathrm{i} = 0$$
$$w^2 - 3w + 3 + \mathrm{i} = 0$$
$$w = \frac{3 \pm \sqrt{9 - 4(3 + \mathrm{i})}}{2}$$
$$= \frac{3 \pm \sqrt{-3 - 4\mathrm{i}}}{2}$$
$$= \frac{3 \pm (1 - 2\mathrm{i})}{2}$$

GC is allowed in this question. Hence it should be used to evaluate $\sqrt{-3-4\mathrm{i}} = 1 - 2\mathrm{i}$.

$\therefore w = 2 - \mathrm{i}$ or $w = 1 + i$

When $w = 1 + i$, $z = \frac{10}{3} + \frac{5}{3}\mathrm{i}$

or

When $w = 2 - \mathrm{i}$, $z = \frac{14}{3} - \mathrm{i}$.

PAIRWISE solutions for w and its corresponding z should be shown clearly for the final answer!

Question 2
Level of Difficulty: Medium
General Comments:

(i) Poorly attempted by majority. Many students treated x and t as constants when differentiating y wrt x or t instead of using implicit differentiation.

(ii) This part was generally well done for those attempted. Common mistakes:
- assume that $Ct = D$
- when $t = 1, y = k$ or $t = 0, y = k$

Many students did not simplify the final answer to obtain a single logarithmic function.

(iii) This part was omitted by majority. About 10 students obtained the correct answer for the initial population.

Qn	Solution
2	**Differential Equation**
(i)	$y-k=xt$ Differentiate w.r.t t, $\Rightarrow \dfrac{dy}{dt}=t\dfrac{dx}{dt}+x$, substitute into the given DE, $t\dfrac{dx}{dt}+x=\dfrac{3t^2+xt^2+2xt}{t(t+2)}$ $t\dfrac{dx}{dt}=\dfrac{3t^2+xt^2+2xt-xt(t+2)}{t(t+2)}=\dfrac{3t^2+xt^2+2xt-xt^2-2xt}{t(t+2)}$ $t\dfrac{dx}{dt}=\dfrac{3t^2}{t(t+2)}\Rightarrow\dfrac{dx}{dt}=\dfrac{3}{t+2}$ (shown)
(ii)	$\dfrac{dx}{dt}=\dfrac{3}{t+2}$ $x=\int\dfrac{3}{t+2}\,dt=3\ln\|t+2\|+C,\quad C\in\mathbb{R}$ $\dfrac{y-k}{t}=3\ln\|t+2\|+C$ $y=k+3t\ln\|t+2\|+Ct$ $=k+3t\ln(t+2)+Ct,\quad \because t+2>0$ When $t=12, y=k$ into $y=k+3t\ln(t+2)+Ct$ $k=k+3(12)\ln 14+C(12)$ $C=-3\ln 14$ Therefore, $y=k+3t\ln(t+2)-(3\ln 14)t$ $\Rightarrow y=k+3t\ln\left(\dfrac{t+2}{14}\right)$

(iii)	

Question 3
Level of Difficulty: Medium
General Comments: This question was generally not well done.

(i) There were many students who did not know what to do after combining the two terms, $\frac{r^2}{2^r}-\frac{(r+1)^2}{2^{r+1}}$, or after getting the quadratic expression, r^2-2r-1.

(ii) A handful of students did not use part (i) to solve the question. Instead, they incorrectly substituted the numbers into $\frac{(r-1)^2}{2^{r+1}}-\frac{1}{2^r}$ and found themselves stuck.

Several students got the final expression in terms of u, u_1-u_{n+1}. However, they incorrectly substituted $\frac{(r-1)^2}{2^{r+1}}-\frac{1}{2^r}$ as the expression for u_r.

(iii) This part was very badly attempted. Many students forgot to change the limits after replacing and several did not realize that a geometric series was involved. Presentation was extremely messy and difficult to comprehend as several steps were omitted.

Qn	Solution
3	**MOD, Summation, APGP**
(i)	$u_r-u_{r+1}=\frac{r^2}{2^r}-\frac{(r+1)^2}{2^{r+1}}$ $=\frac{2r^2-r^2-2r-1}{2^{r+1}}$ $=\frac{r^2-2r-1}{2^{r+1}}$ $=\frac{(r-1)^2-2}{2^{r+1}}$ $=\frac{(r-1)^2}{2^{r+1}}-\frac{1}{2^r}$
(ii)	$\sum_{r=1}^{n}\left(\frac{(r-1)^2}{2^{r+1}}-\frac{1}{2^r}\right)=\sum_{r=1}^{n}(u_r-u_{r+1})$ $=u_1-\cancel{u_2}$ $+\cancel{u_2}-\cancel{u_3}$ $+\cancel{u_3}-\cancel{u_4}$ $+\cancel{u_4}-u_5$ $\vdots$ $+u_{n-1}-\cancel{u_n}$ $+\cancel{u_n}-u_{n+1}$ $=u_1-u_{n+1}$ $=\frac{1}{2}-\frac{(n+1)^2}{2^{n+1}}$

(iii)

$$\cancel{\sum_{r=1}^{n} \frac{r^2-2r+1}{2^{r+1}} = \sum_{r=1}^{n} \frac{2r^2}{2^r} + \sum_{r=1}^{n} \frac{-2r+1}{2^{r+1}}}$$

$$\sum_{r=1}^{n} \frac{(r-1)^2}{2^{r+1}} = \frac{1}{2} - \frac{(n+1)^2}{2^{n+1}} + \sum_{r=1}^{n} \frac{1}{2^r}$$

$$= \frac{1}{2} - \frac{(n+1)^2}{2^{n+1}} + \frac{\frac{1}{2}}{1-\frac{1}{2}}\left(1-\left(\frac{1}{2}\right)^n\right]$$

$$= \frac{3}{2} - \frac{(n+1)^2}{2^{n+1}} - \frac{1}{2^n}$$

$$= \frac{3}{2} - \frac{(n+1)^2}{2^{n+1}} - \frac{1}{2^n}$$

Sub $r = (r-1)$ into r

$$\sum_{r=1}^{n} \frac{r^2}{2^r} \rightarrow \sum_{r=2}^{n+1} \frac{(r-1)^2}{2^{r-1}} = \sum_{r=2}^{n+1} \frac{4(r-1)^2}{2^{r+1}}$$

$$= 4\sum_{r=2}^{n+1} \frac{(r-1)^2}{2^{r+1}}$$

$$= 4\left(\frac{3}{2} - \frac{(n+2)^2}{2^{n+2}} - \frac{1}{2^{n+1}}\right)$$

$$= 6 - \frac{(n+2)^2}{2^n} - \frac{1}{2^{n-1}} \text{ (shown)}$$

Question 4
Level of Difficulty: Medium
General Comments:

Only about 10 students got full marks for this question, mainly due to (iii).

Part (i) was well-attempted by most students. Students who re-expressed the equation as $\left(1+\sin^2 x\right)y=\sin 2x$ and used product rule were most successful although some had difficulty differentiating $\sin^2 x$ and had to first convert to $\frac{1-\cos 2x}{2}$ using double-angle formula. Note that differentiating $\sin^2 x$ wrt x is a basic skill all students should know. Students should avoid using product rule with negative power or quotient rule in order to simplify working. Students who tried to prove LHS = RHS were the least successful.

Part (ii) was also relatively well-attempted although there were some careless mistakes calculating the values resulting in the wrong answer. Students should try to consolidate terms (provided you don't make careless mistakes) in order to simplify the working since the number of terms double after each differentiation when using product rule.

Part (iii) was very poorly attempted. Most candidates were not able to see the link between the parts. ECF from (ii) to (iii) is at most one mark for those who managed to see the link but obtained the wrong answer due to errors in (ii). Students who did "$\ln\left(1+\sin^2 x\right)=\ln\left(\sin 2x\right)-\ln\left(y\right)$" **gained no credit** because the expression for y in (ii) simply does not have sufficient terms to obtain an expansion up to x^4 for (iii).

Qn	Solution	
4	**Maclaurin's Series, Binomial Theorem + Integration**	
(i)	$y=\frac{\sin 2x}{1+\sin^2 x}$ $\left(1+\sin^2 x\right)y=\sin 2x$ Differentiate wrt. x. $\left(2\sin x\cos x\right)y+\left(1+\sin^2 x\right)\frac{dy}{dx}=2\cos 2x$ $\left(\sin 2x\right)y+\left(1+\sin^2 x\right)\frac{dy}{dx}=2\cos 2x$	Students should avoid using product rule with negative power or quotient rule in order to simplify working. "Prove": steps need to be shown clearly to get full credit
(ii)	Differentiate wrt. x. $\left(2\cos 2x\right)y+\sin 2x\frac{dy}{dx}+\left(2\sin x\cos x\right)\frac{dy}{dx}+\left(1+\sin^2 x\right)\frac{d^2y}{dx^2}=-4\sin 2x$ $\left(2\cos 2x\right)y+2\left(\sin 2x\right)\frac{dy}{dx}+\left(1+\sin^2 x\right)\frac{d^2y}{dx^2}=-4\sin 2x$ Differentiate wrt. x.	Consolidate similar terms if possible (but be careful not to make careless mistakes)

$$-2(2\sin 2x)y+2\cos 2x\frac{dy}{dx}+2\left[2\cos 2x\frac{dy}{dx}+\sin 2x\frac{d^2y}{dx^2}\right]$$

$$+2\sin x\cos x\frac{d^2y}{dx^2}+\left(1+\sin^2 x\right)\frac{d^3y}{dx^3}=-8\cos 2x$$

$$(-4\sin 2x)y+(6\cos 2x)\frac{dy}{dx}+(3\sin 2x)\frac{d^2y}{dx^2}+\left(1+\sin^2 x\right)\frac{d^3y}{dx^3}=-8\cos 2x$$

When $x=0$,
$y=0$

$\frac{dy}{dx}=2$

$\frac{d^2y}{dx^2}=0$

$\frac{d^3y}{dx^3}=-20$

$$y=0+2x+0x^2+\frac{(-20)}{3!}x^3+\ldots$$

$$=2x-\frac{10}{3}x^3+\ldots$$

1. Simplify coefficients
2. Presentation:

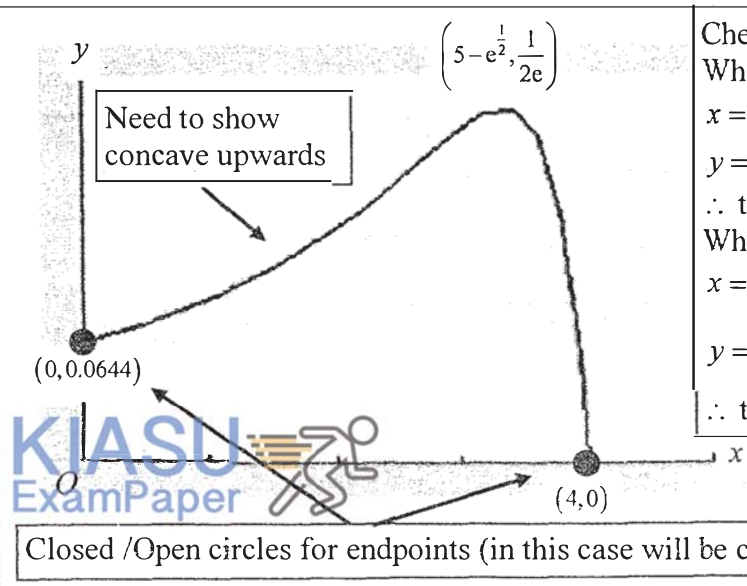
$y=2x-\frac{10}{3}x^3+\ldots$ or

$y\approx 2x-\frac{10}{3}x^3$

(iii)

Question 5
Level of Difficulty: Medium
General Comments:

- Part **a(i)**, is generally well done, with most mistakes due to carelessness or differentiate wrongly
- Most students are not able to get full credit from the graph in part **a(ii)**, due to missing out the y-intercept
- Students will not get full marks for part **a(iii)** if their $\frac{dy}{dx}$ from part **a(i)** is incorrect, even though the final answer may be the same

Qn	Solution
5	**Differentiation & Integration**
(a) (i)	$\frac{dx}{dp}=-e^p$ & $\frac{dy}{dp}=\frac{e^{2p}-p\left(2e^{2p}\right)}{e^{4p}}=\frac{1-2p}{e^{2p}}$ Remember chain rule $\frac{dy}{dx}=\frac{dy}{dp}\div\frac{dx}{dp}=\left(\frac{1-2p}{e^{2p}}\right)\left(\frac{1}{-e^p}\right)=\frac{2p-1}{e^{3p}}$ At stationary point, $\frac{dy}{dx}=0$ $\frac{2p-1}{e^{3p}}=0$ $\Rightarrow p=\frac{1}{2}$ When $p=\frac{1}{2}$, $x=5-e^{\frac{1}{2}}$ & $y=\frac{1}{2e}$ Coordinates of the stationary point are $\left(5-e^{\frac{1}{2}},\frac{1}{2e}\right)$.
(ii)	 Check the endpoints for p: When $p=0$, $x=5-e^0=4$ $y=0$ $\therefore$ there is a x-intercept When $p=\ln 5$, $x=5-e^{\ln 5}=0$ $y=\frac{\ln 5}{e^{2\ln 5}}=\frac{\ln 5}{25}=0.0644$ (3 s.f.) $\therefore$ there is a y-intercept

(iii)	When $p=1$, $\frac{dy}{dx}=\frac{2-1}{e^3}=\frac{1}{e^3}$ $x=5-e$ & $y=\frac{1}{e^2}$ Equation of tangent: $y-\frac{1}{e^2}=\frac{1}{e^3}\left(x-(5-e)\right)$ ← Formula for line $y-y_1=m(x-x_1)$ where m is gradient and (x_1,y_1) is a point on line $y=\frac{1}{e^3}x-\frac{5}{e^3}+\frac{1}{e^2}+\frac{1}{e^2}$ $y=\frac{1}{e^3}x-\frac{5}{e^3}+\frac{2}{e^2}$ $\therefore A=1, B=-5, C=2$
(b)(i)	$x=5-e^p \Rightarrow e^p=5-x$ $p=\ln(5-x)$ since $e^p>0$ — Must show $p=\ln(5-x)$ $y=\frac{\ln(5-x)}{(5-x)^2}$
(ii)	Volume of solid obtained $=\pi\int_0^4\left(\frac{\ln(5-x)}{(5-x)^2}\right)^2 dx=0.2002$ units^3 — Just use G.C., as the questions asks for "4 significant figures"

Question 6
Level of Difficulty: Medium
General Comments:
(a) (i) It is recommended to draw a diagram to represent the given ratio. Many students got the incorrect answer because of the misinterpretation of the ratio given. $AC = 2CB$ means that AC is twice the length of CB.
(ii) Modulus sign is required for the dot product when finding length of projection. It can only be removed if it is clearly shown that the dot product is positive.

Presentation for vectors:
- Write vector as $\underset{\sim}{a}$ not a.
- Use $\times$ to represent cross product in vectors and not general multiplication to avoid confusion.

Qn	Solution
6	**Vectors**
(a)(i)	Using ratio theorem, $\overrightarrow{OC} = \frac{1}{3}(\mathbf{a}+2\mathbf{b})$.
(a)(ii)	Length of projection of OC onto OB $= \dfrac{\left\|\overrightarrow{OC}\cdot\overrightarrow{OB}\right\|}{\left\|\overrightarrow{OB}\right\|}$ $= \dfrac{\left\|\frac{1}{3}(\mathbf{a}+2\mathbf{b})\cdot\mathbf{b}\right\|}{\|\mathbf{b}\|}$ $= \dfrac{1}{3\|\mathbf{b}\|}\left\|\mathbf{a}\cdot\mathbf{b}+2\|\mathbf{b}\|^2\right\|$ $= \frac{1}{6}\left\|(1)2\cos 60^\circ + 2(4)\right\|$ $= \frac{1}{6}\left\|2\left(\frac{1}{2}\right)+2(4)\right\|$ $= \frac{3}{2}$ (Shown) Remember the modulus sign! Use of brackets is important. $\frac{1}{3}(\mathbf{a}+2\mathbf{b})\cdot\mathbf{b}$ is not the same as $\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}\cdot\mathbf{b}$ $\overrightarrow{ON} = \left\|\overrightarrow{ON}\right\|\hat{\mathbf{b}}$ $= \frac{3}{2}\left(\frac{\mathbf{b}}{2}\right)$ $= \frac{3}{4}\mathbf{b}$ Question says deduce! $\overrightarrow{ON}$ is in fact the projection vector of $\overrightarrow{OC}$ onto $\overrightarrow{OB}$ *Learn to see the LINK*
(b)(i)	$\left\|\overrightarrow{OA}\right\| = \sqrt{\alpha^2+\beta^2+\gamma^2}$ $\therefore \cos\theta = \dfrac{\alpha}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$ $\cos\phi = \dfrac{\beta}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$

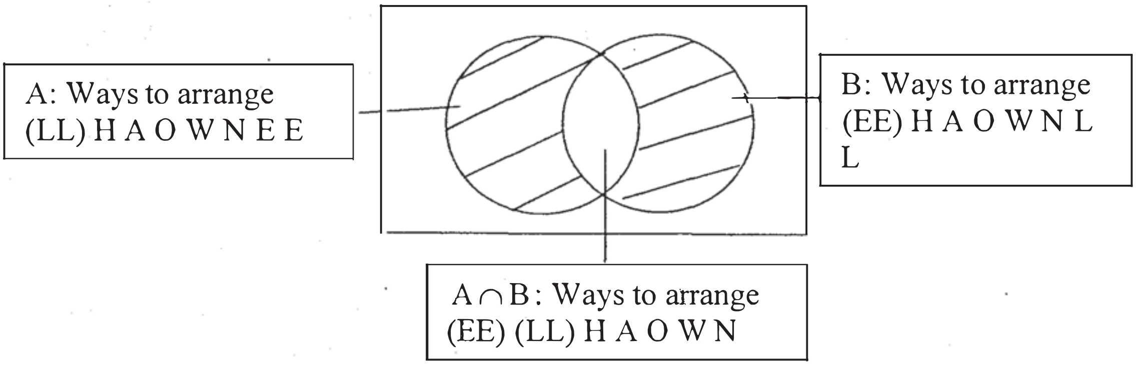
	$\cos\delta = \dfrac{\gamma}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$ $\cos^2\theta+\cos^2\phi+\cos^2\delta$ $=\left(\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2+\left(\dfrac{\beta}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2+\left(\dfrac{\gamma}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2$ $=\dfrac{\alpha^2+\beta^2+\gamma^2}{\alpha^2+\beta^2+\gamma^2}$ $=1$
(b)(ii)	$\cos 2\theta+\cos 2\phi+\cos 2\delta$ $=2\cos^2\theta-1+2\cos^2\phi-1+2\cos^2\delta-1$ $=2\left(\cos^2\theta+\cos^2\phi+\cos^2\delta\right)-3$ $=2(1)-3$ $=-1$ Use of double angle formula & (b)(i) result
(b)(iii)	$\cos^2 45^o+\cos^2 60^o+\cos^2\delta=1$ $\dfrac{1}{2}+\dfrac{1}{4}+\cos^2\delta=1$ Use of (b)(i) result $\Rightarrow\quad \delta=60°$ (Taking acute angle) Also, $\lvert\mathbf{d}\rvert=12$, hence $\alpha=12\cos 45°=12\left(\dfrac{\sqrt{2}}{2}\right)=6\sqrt{2}$ $\beta=12\cos 60°=12\left(\dfrac{1}{2}\right)=6$ $\gamma=12\cos 60°=12\left(\dfrac{1}{2}\right)=6$ $\therefore\quad \mathbf{d}=6\sqrt{2}\mathbf{i}+6\mathbf{j}+6\mathbf{k}$ Question asked for cartesian form

Question 7
Level of Difficulty: Low
General Comments: This question was not well done in general. Most students scored full credit for part **(i)**, but parts **(ii)** and **(iii)** leaves much room for improvement. Students need to read the question carefully and identify if the question is asking for **'number of arrangements'** or **'probability'**.

(ii) Many students attempted to do this part by considering $\frac{8!}{2!}$ as the number of ways to arrange such that 'LL' are adjacent to each other, and similarly for 'EE', without realising that doing so includes the cases where both 'LL' and 'EE' are adjacent, and hence the need to deduct these cases from the numbers calculated.

(iii) This part was poorly attempted, with many students attempting to do this part by considering $5!\binom{6}{4}\left(\frac{4!}{2!2!}\right)$ as the number of ways to arrange the letters, without realising that doing so means that all the letters 'LLEE' will need to be separated, which need not be the case, 'LELEHAOWN' is one such example. Quite a number of students also tried to do this part by splitting into cases, which is an inappropriate approach for this question given the complexity and number of cases that would have to be considered. Students should attempt to use complement method whenever they realise that there are too many cases to consider.

Qn	Solution
7	**Probability (P&C)**
(i)	$\frac{9!}{2!2!} = 90720$ To account for the identical letters 'LL' and 'EE'.
(ii)	Case 1: (LL) H A O W N then slot EE into the 7 possible spaces. Case 2: (EE) H A O W N then slot LL into the 7 possible spaces. $6!\times\binom{7}{2}\times 2 \text{ cases} = 30240$ No. of ways to arrange (LL) H A O W N Choose 2 slots to slot 2'E's out of the 7 slots $\therefore$ required probability $= \frac{30240}{90720} = \frac{1}{3}$ **Alternative:** No. of ways to arrange (LL) H A O W N E E $n(A)$ = Number of ways to arrange with 'LL' adjacent $= \frac{8!}{2!}$ No. of ways to arrange (EE) H A O W N L L $n(B)$ = Number of ways to arrange with 'EE' adjacent $= \frac{8!}{2!}$ $n(A\cap B)$ = Number of ways to arrange with 'LL' adjacent and 'EE' adjacent $= 7!$ No. of ways to arrange (EE) (LL) H A O W N



$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$ $\quad \boxed{n(A \cap B)}$

$$\left[\left(\frac{8!}{2!}\right)+\left(\frac{8!}{2!}\right)-(7!)\right]-(7!)=30240$$

$$\text{Required Probability} = \frac{30240}{90720}$$

(iii)

From **(ii)**

7! = No. of ways to arrange (EE) (LL) H A O W N

$$\text{Required probability} = 1-\frac{1}{3}-\frac{7!}{90720}=\frac{11}{18}$$

Alternative:

No. of arrangements for no two adjacent letters the same

= all possible arrangements without restriction – exactly one pair of identical letters together – the two pairs of identical letters together

$= 90720 - 30240 - 7!$

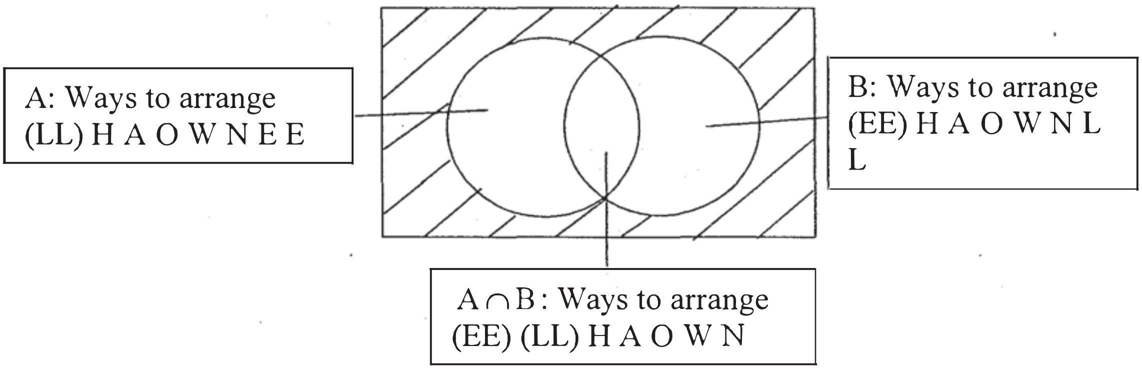
$= 55440$

From **(i)**

No. of ways to arrange (EE) (LL) H A O W N

$$\therefore \text{required probability} = \frac{55440}{90720} = \frac{11}{18}$$

Alternative:



No. of arrangements for no two adjacent letters the same

= all possible arrangements without restriction – (LLs are paired together + EEs are paired together – LLs & EEs are paired together)

$= 90720 - \left(\frac{8!}{2} + \frac{8!}{2} - 7!\right)$ $\quad \boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

$= 55440$

$\therefore$ required probability $= \frac{55440}{90720} = \frac{11}{18}$

Question 8
Level of Difficulty: Medium
General Comments:

- Note that no. of reservations may exceed the capacity allowed in question with context like airline flight booking and hotel room booking.
- When you are answering assumptions/conditions for a Binomial Distribution question, pls answer IN CONTEXT to the question! Also note that the two key assumptions that you should be writing down should be **Independence** of the **event** and **Constant probability** of occurrence for the trials stated in the question. DO NOT MIX the two conditions up.

Qn	Solution
8	**Binomial Distribution**
	Assumptions: 1. Whether a randomly chosen customer does not show up on a day is independent of any other customers. 2. The probability that a customer does not show up is constant for every customer.
(i)	Note: X is the no. of customers, not the no. of rooms. Read the question carefully for hints: 1. "State two assumptions needed such that the **number of customers who do not show up at the hotel** may be well-modelled by binomial distribution". 2. The probability p described in the question is "probability that a **customer with a hotel reservation will not show up**"
(ii)	

Question 9
Level of Difficulty: Low
General Comments:
Part (i) and (ii)

- Students do know that it is a question on discrete random variables (DRV) and most attempted this question and have done well
- A number of students forgot to calculate $P(X=0)$, which is a rather big problem as students should know that total probability for a probability distribution table is equal to 1.
- Most students who managed to obtain the probability distribution got the expectation and variance correct.

Part (iii)

- For the conditional probability, the given condition is that the total number of points is three, hence there are two cases when this happens: Case 1- two red and one blue. Case 2- three red.

Qn	Solution
9	**Probability and DRV**
(i)	$P(X=1)=3\left(\frac{8}{11}\right)\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)=\frac{28}{55}$ Using Probability $P(X=0)=\left(\frac{8}{11}\right)\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)=\frac{56}{165}$ $P(X=2)=3\left(\frac{8}{11}\right)\left(\frac{3}{10}\right)\left(\frac{2}{9}\right)=\frac{8}{55}$ $P(X=3)=\left(\frac{3}{11}\right)\left(\frac{2}{10}\right)\left(\frac{1}{9}\right)=\frac{1}{165}$ Total probability is equal to 1. <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X=x)$</td><td>$\frac{56}{165}$</td><td>$\frac{28}{55}$</td><td>$\frac{8}{55}$</td><td>$\frac{1}{165}$</td></tr></table>
(ii)	Using GC, $E(X)=\frac{9}{11}$, $\text{Var}(X)=\frac{288}{605}$ Alternative: $E(X)=0\left(\frac{56}{165}\right)+1\left(\frac{28}{55}\right)+2\left(\frac{8}{55}\right)+3\left(\frac{1}{165}\right)=\frac{9}{11}$ $E(X^2)=0^2\left(\frac{56}{165}\right)+1^2\left(\frac{28}{55}\right)+2^2\left(\frac{8}{55}\right)+3^2\left(\frac{1}{165}\right)=\frac{63}{55}$ $\text{Var}(X)=E(X^2)-[E(X)]^2$ $=\frac{63}{55}-\left(\frac{9}{11}\right)^2$ $=\frac{288}{605}$

(iii)	$P(\text{exactly 2 red balls selected} \mid \text{total number of points is 3})$ $= \dfrac{P(\text{exactly 2 red balls selected} \cap \text{total number of points is 3})}{P(\text{total number of points is 3})}$ $= \dfrac{P(X=2)}{P(X=2)+P(X=3)}$ $= \dfrac{\frac{8}{55}}{\frac{8}{55}+\frac{1}{165}} = \dfrac{24}{25}$

Question 10
Level of Difficulty: Low
General Comments:

- There is quite a huge disparity between students for this question. There were many students who scored full marks, though presentation could be improved. Similarly, there were many students who do not even know how to start. This is shocking because this question is inspired by past A level question 2007/II/Q8 and seen in tutorial.
- Students should take note that presentation (especially for Statistics) is very important and can affect the meaning of what you are writing, even if you have all the right values. Take note of the common mistakes shown below. Always start by defining any variables you use and the parameters.

Note: Marks are not awarded for wrong equations leading to the correct numerical answers.

Qn	Solution
10	**Normal and Sampling Distribution**
(i)	**Method 1: Working with price**

Let X be the selling price of salmon

$X \sim \mathrm{N}\left(17\mu, 17^2\sigma^2\right)$

Common mistakes:
1. Not defining variable and distribution.
2. writing variance for selling price as $17\sigma^2$

$\mathrm{P}\left(X < 48.60\right) = 0.24$

Common mistake: Defining X as **mass** of salmon then comparing it with 48.60 (which is selling **price**)

$\Rightarrow \mathrm{P}\left(Z < \dfrac{48.60 - 17\mu}{17\sigma}\right) = 0.24$

$\Rightarrow \dfrac{48.60 - 17\mu}{17\sigma} = -0.70630$

Common mistakes:
1. Using wrong standardisation formula, e.g. $\dfrac{48.60-17\mu}{17^2\sigma^2}$ and $\dfrac{17\mu - 48.60}{17\sigma}$.
2. Using wrong mean and s.d. during standardisation. E.g. $\dfrac{48.60-17\mu}{17^2\sigma^2}$

$\Rightarrow 17\mu - 12.0071\sigma = 48.60 \quad ----(1)$

$\mathrm{P}\left(X > 67.50\right) = 0.3$

$\mathrm{P}\left(Z > \dfrac{67.50 - 17\mu}{17\sigma}\right) = 0.3$

$\Rightarrow \dfrac{67.50 - 17\mu}{17\sigma} = 0.52440$

$\Rightarrow 17\mu + 8.9148\sigma = 67.50 \quad ----(2)$

Using G.C., $\mu = 3.5$ (shown) and $\sigma = 0.9$ (1 d.p.)

Read question carefully! It is to be rounded off to 1 d.p. and not 3 s.f.

Method 2: Working with mass

Let S be the mass of salmon in kg

$S \sim \mathrm{N}\left(\mu, \sigma^2\right)$

$\mathrm{P}\left(17S < 48.60\right) = 0.24$

$\mathrm{P}\left(S < 2.8588\right) = 0.24$

Common mistakes: using wrong standardisation formula, e.g. $\dfrac{2.8588-\mu}{\sigma^2}$

$\Rightarrow \mathrm{P}\left(Z < \dfrac{2.8588 - \mu}{\sigma}\right) = 0.24$

$\Rightarrow \dfrac{2.8588 - \mu}{\sigma} = -0.70630$

$\Rightarrow \mu - 0.70630\sigma = 2.8588 \quad ----(1)$

$\mathrm{P}\left(17S > 67.50\right) = 0.3$

$\mathrm{P}\left(S > 3.9706\right) = 0.3$

$\Rightarrow \mathrm{P}\left(S < 3.9706\right) = 1 - 0.3 = 0.7$

$\Rightarrow \mathrm{P}\left(Z < \dfrac{3.9706 - \mu}{\sigma}\right) = 0.7$

	$\Rightarrow \dfrac{3.9706-\mu}{\sigma}=0.52440$ $\Rightarrow \mu+0.52440\sigma=3.9706 \quad ----(2)$ Using G.C., $\mu=3.5$ and $\sigma=0.9$ (1 d.p.) (shown)
(ii)	Let Y be the total selling price of a randomly chosen salmon and a randomly chosen mackerel $Y \sim \mathrm{N}\left(17[3.5]+15[1.2],17^2\left[0.9^2\right]+15^2\left[0.3^2\right]\right)$ $\therefore Y \sim \mathrm{N}(77.5,254.34)$ $\mathrm{P}(Y>95)=0.13625$ $=0.136 \quad (3\text{ s.f.})$

Note that you have to **<u>state the distribution</u>** and the relevant parameters! Calculating only E(Y) and Var (Y) is not enough because that does not automatically mean that X follows normal distribution!

Despite having the correct values of mean and variance, final value is still wrong because you forgot to square root variance in GC for the value of s.d. in normalcdf.

Question 11
Level of Difficulty: Medium
General Comments:

- The values for all the unbiased estimates are **exact** for this question, and must not be rounded off to 3.s.f
- Part (iv) is very similar to Hypothesis Tutorial Question 5, but the combination of samples proved confusing to many students. Common mistakes included using the second sample of 50 only to find unbiased estimates, adding the unbiased estimates of both samples together or applying some inappropriate combination technique.
- For Part (iv), **the question requires the p-value only**. Many students performed the entire hypothesis test and provided a lengthy conclusion which was unnecessary.
- Symbols for the hypothesis test were also poorly written. There are differences between
  - $\overline{X}$ (a distribution) and $\overline{x}$ (a value),
  - X (number of hours) and $\overline{X}$ (corresponding sampling distribution)
- A sizeable number of students said that X is normal, but the question did not state so. Thus, Central Theorem Limit must be used so that $\overline{X}$ will be normal.

Qn	Solution
11	**Hypothesis Testing**
(i)	Let X be the number of hours a randomly chosen gamer spends on the new game daily (in hours). Let μ denote the population mean time spent on the new game daily by gamers (in hours). $\overline{x} = \frac{\sum x}{n} = \frac{126}{30} = 4.2$ $s^2 = \frac{1}{29}\left(550.1525 - \frac{126^2}{30}\right) = 0.7225$
(ii)	H_0: $\mu = m$ H_1: $\mu < m$ Determining whether the time spent has been overstated by the company, hence the alternative hypothesis is H_1: $\mu < m$. Common mistake (Poor presentation) $H_0 = m$, $H_1 < m$
(iii)	Under H_0, Since $n = 30$ is large, by Central Limit Theorem, $\overline{X} \sim N\left(m, \frac{0.7225}{30}\right)$ approximately NOT CLT NOT 4.2 Test Statistic: $Z = \frac{\overline{X} - m}{\sqrt{\frac{0.7225}{30}}}$ Level of significance: 5% Reject H_0 if Since H_0 is not rejected,

(iv)	Let W be the number of hours a randomly chosen gamer in the combined sample of 80 spends on the new game daily. $\sum w = 126 + 215 = 341$ $\sum w^2 = 550.1525 + 964.19 = 1514.3425$ $\bar{w} = \frac{\sum w}{n} = \frac{341}{80} = 4.2625$ $s^2 = \frac{1}{79}\left(1514.3425 - \frac{341^2}{80}\right) = 0.77$ H_0: $\mu = 4.5$ H_1: $\mu \neq 4.5$ Level of significance: 5% Common Mistake Unbiased estimate of population mean for the combined sample is $\bar{w} = \frac{126}{30} + \frac{215}{50} = 8.5$ Under H_0, $p-$value $= 0.015485$ (to 5 s.f.) $= 0.0155$ (to 3 s.f.)